Short Meeting N. Y., March 28. General Meeting Phila., May 16-18.

Entered as matter of the second class at the New York, N. Y. Post Office, November 26th, 1890.

VOLUME XVII.
NUMBER 1.

JANUARY, 1900.

Annual Subscription $5.00
Single Copies . .50

TRANSACTIONS

OF THE

AMERICAN INSTITUTE

OF

ELECTRICAL ENGINEERS.

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PUBLISHED MONTHLY BY THE INSTITUTE, UNDER THE SUPERVISION OF THE COMMITTEE ON PAPERS AND MEETINGS.

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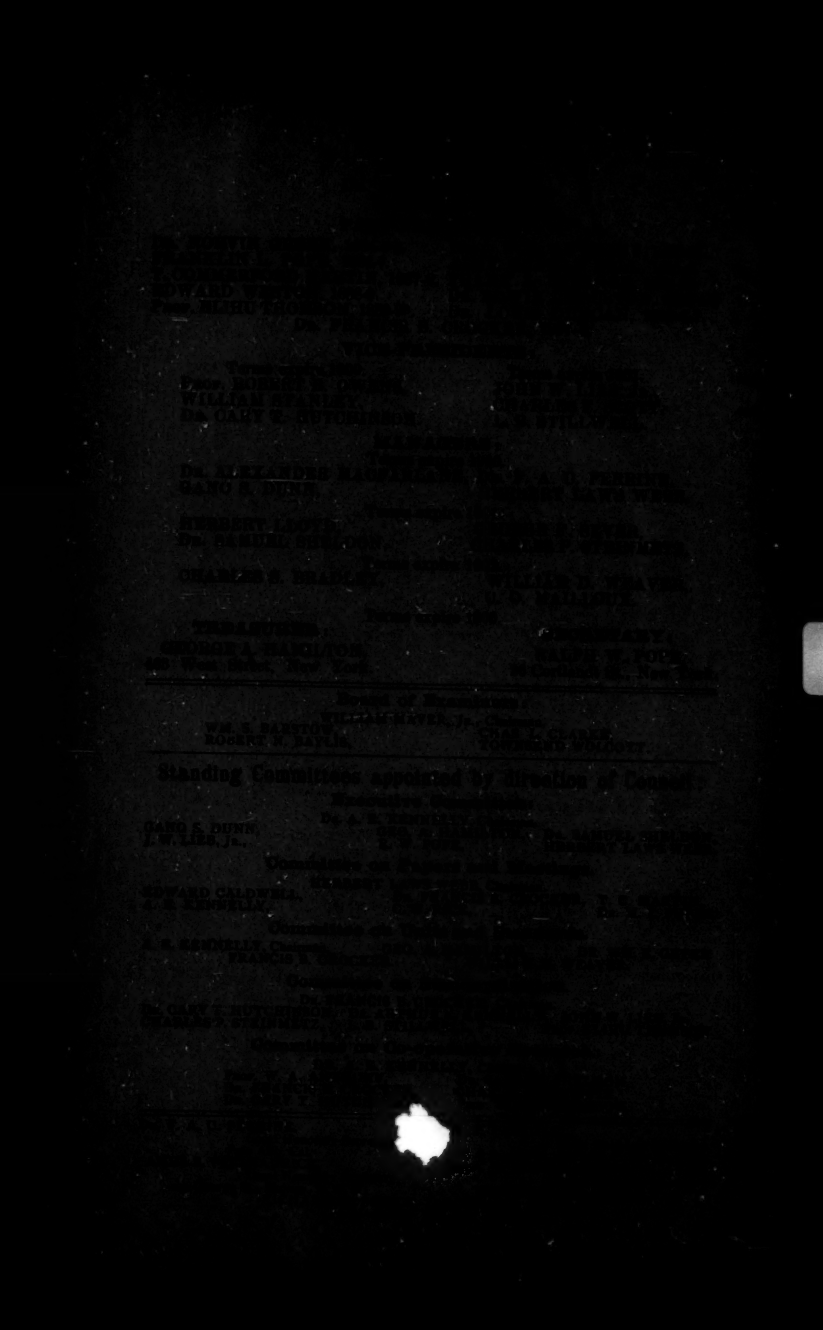

A paper presented at the 190th Meeting of the American Institute of Electrical Engineers, New York and Chicago, February 28th, 1900.

[ADVANCE COPY—SUBJECT TO REVISION.]

NOTES ON ELECTRIC TRACTION UNDER STEAM RAILWAY CONDITIONS.

BY EDWARD C. BOYNTON.

On many of the large steam railroad systems in the United States, there are certain sections which present the most favorable conditions for the substitution of electricity for steam as a motive power. These conditions are the result of increasing density of population, and mean that better and cheaper transportation facilities are needed by the public than are provided by the steam road.

The electric street railroads quickly took advantage of these conditions, and by building lines more or less parallel to the steam roads, soon acquired a large share of the local passenger traffic. The fault with the steam road was not that the motive power was steam, but the fare was too high and the train service too infrequent. The whole question of the substitution of electricity for steam hinges upon that one point.

In order to provide satisfactory transportation facilities, the steam road must double or quadruple the number of its trains, and reduce the fare to at most one cent per mile. When there is sufficient density of population, this will surely cause a large increase in the number of passengers carried. This increase is due principally to the fact that many people who could seldom afford the expense of traveling would then make frequent trips. It is very doubtful whether the greater number of steam trains can be operated at a sufficient profit with the low fare. Here, then, comes in the change in motive power, with the sole purpose of decreasing the operating expense.

I wish to call attention to two classes of local passenger traffic

which should be considered as distinct from each other. The suburban traffic of a large city is well understood, and its characteristics are usually such as would make the change from steam to electricity profitable. The low fare will induce a part of the population to make their homes in the suburbs, and thus increase the travel. But where there are competing trolley lines, the steam road, which we will suppose has electric motive power, needs one more facility than those mentioned, and that is, high speed. Without that, there would be little advantage over the competing lines.

It has been proved by experience that the speed must be at least as great as an average steam train, and there is no doubt that if the speed be made as high as the fastest steam express train, the popularity of the line would increase. It is well-known that the business man who desires to travel from one city to another, or to and from his residence and his place of business, cannot be carried there too rapidly. It would probably surprise the average passenger on one of the fast steam express trains to be told at a certain time that he was traveling 70 miles per hour, and yet such speeds are reached every day, for short distances, over a straight, level track.

The other class of traffic referred to is that existing between cities and towns in close proximity. Let us assume a case as an example.

In a certain densely populated manufacturing state, there is a city larger than any other within fifty miles radius. Within that radius are several towns and small cities not over twenty to thirty miles from the larger city. These are connected by the steam road, which maintains what is considered a reasonable train service, and one that is as frequent as the traffic seems to demand, at the rate of fare charged, which is from two to two and one-half cents per mile. The trains are quite heavy, nearly always fully loaded, and are run from two to three hours apart. Together with its freight traffic, such a road pays well, judged from the steam road's standpoint. Suppose that electricity be substituted for steam in that section, and a train service consisting of two, or three car trains running every half hour from each end, with a maximum speed of fifty to sixty miles per hour, and the fare reduced to one cent per mile. There is no doubt in the minds of those who have watched the development of such cases, that the increase in traffic and low operating expenses

would result in a far greater profit than was ever earned by that section of the road. It is well-known that such conditions exist on our steam railroads in many localities.

It has been said that the steam roads will begin by equipping their branch lines with electric motive power, and little or nothing is heard of the equipment of the main trunk line. It is necessary to define what is meant by a branch. In a large system some branches are 100 miles long and may be double tracked; others are from six to forty or fifty miles in length. In the assumed case described above, the conditions may exist on one of the large branches or even on the main trunk line, which may have four tracks. It should make no difference in deciding the question of equipping the part of the system which possesses the desired conditions, whether it is on a branch or a part of the main line. It should be fully understood that no steam railroad will equip any portion of its lines, except with the provision that nothing shall be done which will prevent the running of steam and electric trains over the same track.

A well known authority, nearly five years ago, mentioned the possibility of the equipment of one or two of the tracks of a four-track trunk line by electricity, to carry the local traffic, and stated that the two tracks equipped should be those used by the freight trains. At the present day that does not seem advisable, for the reason that the speed of the electric trains must be equal to that of the steam express trains, and the slow moving freight trains would seriously interfere with the electric schedule. But it is unlikely that any steam road will equip a part of its main trunk line until it has satisfied itself, as to financial results, by giving it a thorough trial elsewhere.

The reduction of fares combined with the use of open cars during the hot summer months, produce a class of passengers which formerly used the electric street cars. These have been called the pleasure riders, and they furnish a considerable proportion of the receipts. Experience with open car trains has shown that speeds of 30 to 35 miles per hour are the maximum which should be used, on account of the discomfort caused by the wind pressure created by the train.

The Question of Equipment.

The questions, how much will it cost to equip a given service to be operated by electricity?—and how much will it cost to

operate it? are frequently asked. The electrical engineer is now in a position to answer both these questions with great accuracy. The experimental stage has passed, and sufficient data is at hand to give all the information needed. It must be realized that the operation of a steam railway by electric power introduces many conditions which do not exist in the transportation problem within a great city, such as are operated by the elevated or surface street railroads. There are no restrictions on speed or weight of trains. Rapid acceleration is not of so much importance, for the stops are much further apart. The trains must be operated under steam rules absolutely, and the whole equipment must comply with the laws relating to steam railway trains. The railway company contemplating the equipment of a part of its system with electric motive power has the choice of several methods which should be closely studied to determine which is best suited for the service it is proposed to operate.

These methods are:

First: The purchase of electric locomotives of sufficient power and weight to haul its standard passenger coaches.

Second: The equipment of a number of its standard coaches as motor cars.

Third: The purchase or building of a sufficient number of special light passenger coaches, some of which are equipped as motor cars, and the withdrawal of its standard coaches entirely from this service.

Fourth: Shall freight be hauled by electricity or steam?

The use of electric locomotives for the purpose under consideration depends upon several conditions. If the travel is heavy, that is 2,000,000 passengers per year and upward, the service frequent, the speed high, requiring an average train of four cars, and, as may be the case, the same coaches must go much further than the electric service extends, hauled by steam, it is advisable to use the electric locomotives hauling standard coaches. Their principal advantages lies in their ability to perform the work of a steam locomotive in every respect, and this is frequently a strong point in their favor with the railway managers. They are thus able to accommodate themselves to congested traffic which usually occurs on holidays and possibly at certain times every day, by simply increasing the number of coaches hauled as is the practice with steam locomotives. Such locomotives should weigh from 100,000 to 150,000 pounds, should have

eight wheels and four motors, so that the total weight is available for traction. They must be provided with sufficient power to haul at least double the average train without over-heating. They must not only be able to perform the work of a steam locomotive in the same service, but should do it at a faster schedule speed. The rapid acceleration of a train hauled by such a locomotive enables it to perform the above duty without any increase in the maximum speed. In switching cars, the ease and rapidity with which the electric motor can be handled is a great advantage.

It is necessary to equip these electric locomotives with the best automatic air brake system that can be obtained, for several reasons. They must operate the existing brake system on the coaches as well as the steam locomotive does. The law requires automatic brakes and a whistle. An independent motor compressor with a large main reservoir is therefore almost imperative.

The cost of repairs on an electric locomotive should be exceedingly low, possibly 10% of that required by a steam locomotive on account of the fewer moving parts and the entire absence of the boiler and its necessary equipment.

The fact that an electric locomotive requires but one set of controlling and air braking apparatus is a distinct advantage over other methods of employing electric motive power. This is evident, not only in the first cost but in the fewer parts to be cared for.

The second method of applying electric motive power to an existing steam railway; the equipment of standard coaches as motor cars, will appeal to all steam railway managers as the cheapest and most convenient way to make the change. This method has strong arguments in its favor.

The locomotive carries its own paying load and during the hours of light travel can be run light, without hauling other coaches. A standard coach equipped with two motor trucks and four motors will haul nearly as many coaches as the locomotive above mentioned and will weigh 100,000 lbs. It will easily handle five coaches making a six car train weighing loaded 450,000 lbs. I believe that the power consumed per passenger carried in a train hauled by an electric locomotive will be less than if all the cars were motor cars, whether run singly or in one train.

Let us see exactly what must be done to a standard coach to equip it as a motor.

In most cases the conditions will be found to be such that three or four car trains with a proper schedule will be sufficient to take care of the maximum traffic. This necessitates only two motors for the coach. These should both be mounted on one truck, and this truck complete with motors will have to be purchased and used to replace one of the standard trucks. The motor truck should be built especially for the purpose, a heavy steel truck, 36 to 40-inch, steel tired wheels, brakes of the type that do not require brake beams, springs both elliptic, and equalizer of sufficient strength to support the weight of half the car body with maximum load, and this means all standing room occupied. The size and general design of the axle in the motor truck must be carefully considered. The author does not believe that steel axles 5″ maximum diameter between wheels, are safe. It may be that in calculating their strength and considering the enormous strains which they must withstand, the result appears satisfactory, but experience shows that the excessive vibration at high speeds will cause crystallization of the steel and a high factor of safety must be employed.

The wheel journals should be at least $5\frac{1}{2}$″ x 9″, and the diameter of axle between wheels $6\frac{1}{2}$″, with a larger diameter through the axle gear.

The wheel base cannot well be more than seven feet, on account of the curves, but it is nearly all needed in order to obtain room for motors of sufficient size. The motor should not be supported by the truck frame in any way.

Steel bars should be placed at each side of the motors extending from one axle to the other, and beneath them, just inside the wheels. These should be suspended from lugs on the motor frame by suitable links as near the center line of each axle as possible. The backs of the motors can then be carried on these bars by means of other lugs on the motor frame and springs above and below the lugs. This method of motor suspension is rapidly coming into general use and it has many advantages. If motors should be damaged it is simply necessary to place another pair of wheels and axles with other motors in the same truck frame. The motor cars ride much easier, or practically the same as before they were equipped with motors, due to the fact that the jarring of the motors is not transmitted to the car body.

The motor car must be wired for, and furnished with a suitable number of electric lights and heaters, which is a simple matter. The same thing must be done for the coaches it is proposed to haul. The latter should also be equipped with collecting shoes connected by a wire which terminates at each end in an electric coupler of sufficient capacity to supply the motors on the motor car if necessary. The motor car is supplied with the regular air brake apparatus used by the road and piped as a steam locomotive, except that the car is double ended and requires an engineer's valve, gauge and other necessary parts at each end.

The independent motor air compressor, main and auxiliary reservoirs, car wiring cables, main wires for collecting shoes, rheostats and electric couplers, all go under the car in addition to the standard equipment of a passenger coach.

A small cab should be provided at each end for the motorman preferably inside the car, fitted with a front and side window which can be opened to their full extent. Here are located the air brake valves, the automatic governor and switch for the compressor, the motor controller, main switch, circuit breaker, electric light and heater switches. The bells or gongs, pilots and whistles at each end complete the list.

A few words about car wiring may be of interest here. The most careful work in wiring cars is very essential. The author believes that the causes of nearly all the fires occuring in motor cars can be traced to defective wiring. There is no reason why such wiring can not be made safe. Even if the insulation of wires and cables is of the best, they should be treated as bare wires, as the insurance underwriters say, and unskilled labor does not pay in this part of the work. The maze of iron pipes, rods and braces, under such cars, render it necessary to protect the wires from rubbing or chafing with the utmost care. It must not be taken for granted that the pipes, etc. remain in one position when the car is under headway. The working and straining of the car body, the swing of the trucks and brake rods and compression of springs must be carefully considered, and no care in protecting the wires however great can make them too safe. In trolley cars the removal of the pole from the wire will put out, or render a fire easy to control, but in such cars as are under discussion, I have not yet seen or heard of a method of cutting off the current at or near the contact shoes, though there may be

such. It is evident therefore how helpless a train crew is when called upon to put out a fire caused by a terrific arc under the car. It has been proved in practice that good car wiring practically prevents such accidents. Before leaving the equipment question, I will give some opinions on the performance of the motors.

For such short distances, and the intermittent work required, the modern railway motor seems well adapted. It can be overloaded, 100% frequently, without injury, for a short time. Most types of these motors have but one or two serious faults. One is lack of sufficient ventilation. During hot weather this becomes a serious matter, and one that should be corrected by the manufacturers. The design of the axle bearing, is another. In street car work, the plan of using a gun metal lining in halves, lubricated by grease or oil is fairly satisfactory, but where these bearings each become 15″ long and 6½″ diameter; at speeds of 40 to 60 miles per hour, the conditions are different, and while giving little trouble, the cost of repairs seems unnecessarily high. Most of the motors are similar in design to the ordinary street-railway motor, entirely enclosed by the frame, and intended to run in the mud and slush of a city street. On the other hand a steam roadbed is usually dry and clean, the motors are further above the ground, and water rarely flies above the axle. It would seem therefore that they could be more open on top. It has been found necessary to run through the hot summer months with the large covers over the commutators removed. The construction, and especially the insulation of armatures has reached such a degree of perfection, that a burn-out is seldom heard of in the larger motors.

I have had under my personal observation a considerable number of such motors, some of which have been in service over two years, making a daily mileage of over 300 miles, and have never had one armature burn out. In one case a pair of motors were in service eight months, including a winter. No repairs whatever were made during that time, beyond the ordinary daily inspection, cleaning, and renewal of carbon brushes.

In another case a pair of motors made upward of 100,000 miles after being put into service, the only repairs being renewal of armature and axle brasses, gears and pinions. As is well-known a frequent cause of burn-out in the past has been due to the wear of the armature brasses allowing the armature

to strike the lower pole-pieces. This cause has been eliminated by making the clearance between the pole-faces and the armature slightly greater, possibly at the expense of an increase in weight of copper on the field magnets, and with no apparent loss in efficiency.

The total weight of a 60-foot standard coach equipped as a motor car with two motors will be about 80,000 pounds, without passengers, of which 55,000 pounds is on the drivers or motor truck. The speed of such a motor car running light, if geared sufficiently high, is probably only limited by its weight and the quality of its track and road bed. With a stone ballasted track, 100-pound steel rails, few curves, and those of long radius, a heavy car with the best steel tired wheels should run 100 miles per hour at full speed without difficulty.

I have mentioned a four-motor car consisting of a 60-foot car body weighing 100,000 pounds complete. This represents about 800 H. P. nominal rating of the motors at 650 volts direct current, and this is the maximum H. P. that can be placed under a standard coach, on trucks and wheels which do not necessitate any other changes in the existing standards of steam practice. The motors are capable of exerting double that power for a short time. The total cost of converting a standard coach into a motor car with two motors is about $3,800.

The third method of using electric traction in steam service, that of the use of light motor cars and trailers, built for the purpose, has some advantages. The former coaches can be used elsewhere on the system as are the locomotives. The smaller, lighter cars are cheaper to construct, the wear on the track is less, and there is considerable economy in power. It has been proposed, and no doubt will come, that such cars will run through the principal street of a city, on the existing street car track, before starting on their trip over the steam track. This would necessitate either a trolley wire over the steam track, instead of a power rail, or both collecting shoes and trolley pole on the car. There is no question but that this may prove a great advantage in time. On the other hand, the cars must be used exclusively on the line equipped with electric power. At speeds of 50 to 60 miles per hour which *must* be made in order to compete successfully with the existing parallel trolley lines, the cost of maintenance and repairs due to the excessive vibration will undoubtedly be greater than that with standard coaches.

The economy in power due to the reduction in dead weight hauled is of considerable importance, not only on account of the smaller amount used, but the line conductors can be lighter, greatly reducing their cost, or the system can be extended to longer distances at no more expense for transmitting the power.

In the future there may be a decided tendency to reduce the weight of electric trains. In other words, it will be an attempt to handle a constantly increasing traffic with a lighter equipment in order to haul less dead weight per passenger. The engineer who proposes to introduce such changes must move with the greatest care in order not to save the weight at the expense of strength. It should never be forgotten that the maximum load is "no standing room," that the strains on a car and its trucks running at 60 miles per hour over a steam track which may be made of 70-pound rails with joints none too good, are not to be compared with such as are met with in a city street at low speed. An interesting problem in connection with a similar equipment has arisen within the last few years, and the time is rapidly approaching when it must be solved. It relates to the difference between the wheels of street cars and the steam railroad coaches. Though both use the standard gauge of 4′-8½″ the average street car wheel has a tread 2½″ wide and a flange ⅝″ deep, while the Master Car Builders standard requires a tread 4″ wide and a flange 1½″ deep. The problem is a serious one, for it intimately concerns the safety of the train. The steam railroad people after 50 years experience have settled upon the above standard, and the electrical engineer who chooses to ignore their experience in this, and in many other cases, runs a risk. There should be no doubt that it is unsafe to run the small street car wheels at high speeds over steam railroad tracks, with their present form of frogs and switches; and it is, in fact, impossible to run them on steel rails weighing 90 to 100 pounds per yard on account of the wide spaces in the frogs. The question may be asked: "Is it not safe to run such narrow wheels on the steam track if the latter is kept carefully to gauge, and proper frogs for these wheels are substituted for the existing ones?" If this is done steam trains can no longer run on the road, and, as it is necessary in order to round a curve at high speed with safety, to spread the gauge from ¼″ to ½″, the danger is greatly increased. On the other hand, the Master Car Builders' wheels cannot run on the existing street car tracks in our cities. The

flange is too deep for the frogs, the size of the groove on the inside of the straight rail is larger than the city authorities would sanction, and the outside portion of the tread would in many cases run on the pavement and crush it down to a level with the top of the rail. The only solution of the problem seems to be a compromise wheel with about 3″ tread and 1″ flange. Whether or not this is safe, only time can tell.

The fourth question, that of hauling freight by electric power, should, of course, be decided upon at the time of installation, as it may cause considerable difference in the plans for power stations and line transmission. As the question can only refer to local freight along the line electrically equipped, it is of doubtful importance as applied to the conditions under discussion. If the freight traffic on such lines be sufficiently heavy to necessitate the use of a locomotive for several hours daily during the hours between midnight and morning when there are few if any electric passenger trains in service, it is economy to use an electric locomotive, for it costs but little more to run the power station, if it has been shut down, and the total expense would be somewhat less than that of a steam locomotive. The whole question of transportation of freight by electric power is one which concerns the future more than the present.

When the time arrives that long distances are electrically equipped on our steam railways, then it becomes far more important.

Power Transmission.

Feeders:—The transmission of electric power forms the most important part of the problem of the electrical equipment of a steam road. At the present day our railway motors all require the direct current, and we are therefore limited to its employment in the working conductor. By increasing its voltage from that usually employed to 700 volts, a considerable advantage is at once gained, and without additional expense in motors or generators. Experience has shown that the economical radius of operation of a power station generating such a current and delivered to the line without feeders is from 10 to 12 miles. This refers to a heavy train service with a fairly frequent schedule, and an average load of 500 amperes on each radial line of single track.

By the line or working positive conductor is here meant a steel rail of 90 to 100 pounds per yard, well bonded, and equal in con-

ductivity to about 1,200,000 cm. of copper. The statement, "without feeders," may be wondered at, and a few words of explanation will be necessary.

It should be remembered that the conditions are very different from a street or elevated road. There may be only two, or at most three trains running. The greatest fall in potential occurs when a train is leaving the further end of the line, and this may be somewhat less than $\frac{3}{4}$ of a volt per ampere at that point, while the average efficiency of the line is over 75%. Again even if the loss in the line becomes greater through an attempt to increase the number of trains, or to extend the line, the question of feeders depends almost wholly on the cost of fuel. If the interest on the cost of feeders is greater than the saving in fuel consumption effected by their use, and a satisfactory train schedule can be maintained without them, it cannot be in the interest of economical operation to provide them. It has been said that such feeders in connection with a so-called booster used to overcome the drop in the feeders, and a consequent decrease in their weight, are the most economical. Such an arrangement is undoubtedly cheaper as regards first cost, but I have seen no data showing the cost of maintenance and depreciation of these additional machines, as compared with the cost of a feeder of sufficient weight to perform the work without the booster, and on which there is practically no depreciation. Whichever method is followed, the cost of feeders for such a road will reach many thousands of dollars, and railway managers will make the most rigid investigation of traffic conditions, present and prospective, before deciding upon such an outlay.

The above statement should make intelligible the reason why 10 or 12 miles is considered the maximum radius of operation of a station delivering 700 volts direct current. It is hardly necessary to add that a larger system extending over greater distances should be supplied by multiphase generators, and a high tension transmission line combined with the usual rotary converters, located at suitable points on the system. In the absence of a practicable alternating current railway motor, the above system is the only one—there is no choice.

In regard to the most economical material to use as feeders, the extremely variable prices of both copper and steel, renders it difficult at present to come to a satisfactory decision, but with both metals at what we may call their normal values, steel is

cheaper and more satisfactory. The author believes a proper feeder for 700 volts direct current should be made of flat steel bars about 1″ x 5″ section, two of which are placed side by side, bolted or riveted together with alternated joints and supported on edge in the slotted tops of small posts set in the ground at the side of the road bed, not over two feet high and boxed in. At grade crossings the break in these feeders must be bridged by either an underground or overhead connection. In yards and stations where there is a multiplicity of tracks it will be frequently necessary to carry them overhead for considerable distances. Copper is of course used in such cases.

The Working Conductor—In considering a train service consisting of heavy trains running at the speeds mentioned, the trolley wire as a working conductor will probably not come into general use, although it is used for such a service to-day. The cost of construction, maintenance and depreciation is greater than that of a third rail. It has few advantages, and many disadvantages for such a service. It is now generally conceded that an insulated rail placed close to the track answers all requirements, and the author's experience shows that it is satisfactory. It is difficult to understand, however, why the common form of T-rail is so generally used for this purpose unless it is due to a desire to save money by using up old rails. A more inconvenient cross-section for thorough and efficient bonding could hardly be selected. It will be admitted by all that this conductor should be so bonded that when worked at its full capacity there should be no greater loss at the joints than elsewhere. There are a number of standard commercial forms of rolled steel which are no more expensive than T-rails that are well suited for this purpose. A form that will permit the use of one or more thin copper plates of ample area of contact held at the joints between a steel splice plate and the conductor by heavy pressure obtained by the use of a sufficient number of bolts, is an inexpensive and satisfactory bond. The rule that the bond shall be equal in carrying capacity to the conductor, and the area of contact equal to or greater than the cross-section of the conductor is a safe one to follow.

One question which has been studied with care is of great importance in this latitude, and that is the effect of ice on the contact surface, and how to get rid of it. Many experiments have been tried, and few can be said to have been successful. A fur-

ther possible advantage of the use of some other form of rolled steel might result in the complete elimination of this trouble. I refer to the collecting shoes having a side or under-running contact. This would allow the partial roofing over the conductor by wood, which would thoroughly protect it from the weather.

Ordinary snow-storms and even blizzards do not interrupt the service. I have seen a storm which tied up nearly every wheel in a nearby State, but the electric service was the last to succumb, and even then it was not on account of the conductor rail, or too much snow on the roadbed, but from a train running off an ice choked frog. It has been demonstrated that motor coaches equipped with proper steel brushes for the third rail and snow plows, can go through as much snow as an ordinary passenger locomotive. Their great advantage lies in the fact that they can run through deep snow slowly, due to the enormous torque of the series motors and the absence of reciprocating parts. But when the temperature of the conductor rail is below freezing point, and it begins to rain, as is not infrequently the case, a coating of ice forms on the contact surface which closely resembles enamel. No mechanical method has been found to completely remove this.

As to chemical methods, certain roads can and do use salt or brine. It is not considered advisable to salt the road bed of such a road as is under discussion owing to the danger of leakage should the track become flooded with water. When applied at the right time, an oil which does not solidify at a low temperature is sometimes successful, but the difficulty of applying it to the whole road at the proper time can be appreciated.

Insulation:—The question of insulating the positive rail of a 700-volt grounded circuit has in actual practice been developed to such an extent that the results obtained are remarkable, to say the least. If such methods as are now in use had been proposed ten years ago, they would have been regarded as impracticable.

For years it was the custom to consider the ground a conductor of electricity. It was of course realized that the surface rails must be bonded in some way, but the ground was considered to be a great aid to the rails in returning the current. I do not propose to deny that this is true in a crowded city where there are thousands of tons of iron pipes buried but a short distance

beneath the rails, but can we call this a ground return? My experience shows that the road bed of a steam road consisting of sand, gravel or rock ballast, when dry, is a good insulator, and when wet there is but little difference. A rock-ballasted track in particular needs no insulation whatever except the wooden ties.

I am aware that such a statement may be regarded with doubt, but perhaps it can be made clearer if we take all things into consideration. The road runs through an open country, the soil is of the average composition, some of it wet, but most of it dry. If we stand on a wet spot and place our body in circuit from positive to ground we receive a shock, perhaps of maximum voltage. This would apparently show the ground to be a conductor, but a little thought will prove that it conducted a few milli-amperes only. If we stand in dry earth or on a tie, we feel no shock. But the one test that proves the insulation of such a line is the leakage test. From tests made every night, for over a year, the leakage averages $\frac{1}{2}$ ampere per mile in dry weather to $1\frac{1}{4}$ amperes in wet weather, and I am convinced that nearly all of this is in the underground work necessary at grade crossings and switch points. The above refers to a rail insulated upon creosoted wood blocks attached to the ties. A complete covering of snow, has little or no effect on the leakage. The form of the positive rail may influence the leakage somewhat. For example, the inverted V form acts as a roof to shed water and keeps the contact surface between the block and rail dry. But there is in use several miles of ordinary T-rail as a positive conductor, laid on blocks of wood $1\frac{1}{2}''$ thick attached to the ties, not creosoted, but dipped in an insulating compound. No leakage is noticable here. We can easily understand that if any appreciable amount of the current in amperes should leak through these blocks whether prepared or not, they would burn up. The writer, therefore, believes that such insulation of the positive rail for the current and voltage under discussion is ample, and much expense can be saved by steam roads by its use.

Track Bonding.—One of the most necessary and at the same time expensive parts of the work in changing existing steam roads into an electric line, is the bonding of the service rails. The author believes he has done some of the heaviest bonding in the country, and is of the opinion that there is no satisfactory method of bonding a T-rail at present. When such bonding

costs two dollars per joint, it becomes a very serious matter. Bonding around the angle plate with the bonds about two feet long, is out of the question, for the cost of copper would be too great, and it would be exposed. Riveting the lugs on the bonds through the web of the rail, is not good practice, because to secure sufficient area of contact four holes would have to be drilled in the ends of each rail, which so weakens it as to render it unsafe. The shortest possible bonds should be used under the base of the rail. It requires four one-inch holes in the base of each rail, and we can easily see how unsatisfactory and expensive this is, with four bonds of 300,000 cm. area for each joint of 100 pound steel. In nearly all rail bonds the principal resistance is in the contacts. It is a simple matter to use sufficient copper, but to secure a proper contact is a difficult problem. The bonds must have the utmost flexibility to withstand the vertical motion of the rail ends, and even then many of them will gradually break off strand by strand. What is urgently needed at the present day is a cheap and efficient bond for a T-rail. Such a bond, to be satisfactory, must show no greater fall in potential than an equal length of the rail itself, when the maximum current is flowing through the joint. On account of the fact that the ground is practically of no value in augmenting the conductivity of the return circuit, the entire circuit must be regarded as metallic, and the ground should not enter into any calculations.

Power Stations.—The writer does not propose to enter into the subject of the design and arrangement of machinery in a power station for a steam road, as there are no engineering features which differ from those encountered in such a station intended for a large street railway. An abundance of water and cheap fuel are of course important points. Such power stations can be built for from $80 to $90 per kilowatt, exclusive of the land.

A few words about the amount of power required may be of interest. An important figure is the amount of power delivered at the switchboard per train mile. It eliminates all losses due to resistance of circuit, and current used for air compressors, electric lights and heaters. This figure will vary from four to six kilowatt hours per train mile, reaching its maximum in December and January, due to the longer hours of lighting the cars, the constant use of electric heaters, and the frequent running through snow.

The question of heating a standard coach by electricity is one that should be thoroughly understood. Street car heating is totally different. The public demands the same temperature as is furnished by steam, which is 68° or 70° F. It makes but little difference what heater is used, provided there are enough of them. One may radiate its heat faster than another, and so raise the temperature of the car more rapidly, but it will require, in any case, from 12 to 15 kilowatts of energy for each coach. An ordinary train consisting of a motor car and two coaches weighing 200,000 pounds, will require, at a speed of 35 to 40 miles per hour on a level track, about 125 kilowatts, or about 166 H. P., of which the car alone would consume 75 K. W., if running light. The motor will consume an average energy of four to five kilowatt-hours per train mile, or 40 to 50 watt hours per ton mile. Power can be produced with condensing engines and fuel at about $2.30 per ton, for about .008 ($\frac{8}{10}$ of a cent) per K. W. hour.

Cost of Operation:—It is most desirable in operating a heavy electric service over a railway on which steam trains are also operated, to arrive at a satisfactory conclusion as to the comparative cost of operating each type of train per mile. If an electric service is entirely substituted for one which has been operated by steam, the railroad company is in a position to know accurately the difference in cost of the two systems. But when both are operated over the same tracks the problem becomes very complex. For example, even if we omit the maintenance of the road way, which may be a little higher in an electric service, there are many other items such as salaries of agents, ticket sellers, gatemen, etc., all of which properly belong to the operating department, which must be proportioned between the two services. It may be said that the cost of operating a steam passenger train has been estimated all the way from 30 cents to $1.00 per mile depending upon the length of the train and other conditions which are seldom alike in the different localities. The author cannot go into this subject in detail, but will give a few points of difference between the two services upon which an approximate estimate can be based. A fair average cost of running a steam locomotive, including fuel, when coal is about $2.30 per ton; water, wages, repairs etc. is 22 cents per mile. The average cost of repairs to coaches may be taken at one cent per mile each. The wages of train crew, consisting of a conduc-

tor, baggage master and one brakeman will average .05 per mile making a total of 30 cents. This figure is intended to represent the lowest possible cost of operating a train of only three cars by steam with the understanding that it is kept almost constantly moving for about 9 hours, and covering from 150 to 200 miles. It is well known that a train making but a few miles per day cannot be run at a profit, either by steam or electricity, due to the fact that cost of wages per mile increases rapidly, as the crew has to be paid the same in either case. A great advantage of the electric service may be mentioned here. The above service is all that can be required of one crew and one locomotive, but the motor car can easily make 300 to 400 miles in 18 hours, and as the daily service is in operation at least that long, one motor car does the work of two locomotives. In the operation of a similar three-car train in which one car is a motor car, we will assume the same crew with the addition of a motorman and omit the locomotive. The cost per mile in wages will then become $6\frac{1}{2}$ cents, that of repairs to cars the same as before, 1 cent, maintenance of motors, $\frac{1}{2}$ cent and cost of power delivered to train, 6 cents, making the total cost per train mile, 14 cents for the electric service.

Electric Lighting.

All steam roads, which have introduced electric motive power, will consider the question of lighting their passenger stations and freight houses along the line. It will be found that lighting in this way is very satisfactory and far cheaper than the purchase of gas or electricity from others. For lighting freight sheds, platforms and other outside lights, the simple wiring of the lights in groups of five or six in series, and connected directly between the feeder or working conductor and the service rails, has been found satisfactory. The occasional interruption of the current due to the opening of a circuit breaker will shut off these lights for a few seconds, which makes them inconvenient for indoor lighting. For stations requiring not over 60, 16 c.p. lights, a small storage battery of 58 cells, together with a rheostat and switchboard with the necessary switches and instruments can be installed for about $900.00. By making the rheostat of about 55 ohms resistance and 35 amperes capacity and connecting it in series with the railway current, it can be used to charge the battery.

The battery and rheostat are connected in parallel with the lighting load, and the resistance so regulated that the railway current does the lighting, the battery merely acting as a regulator, charging slightly when the voltage rises and discharging into the lighting circuit when the pressure falls, due to the movement of the trains. This maintains a practically constant voltage of about 120 on the lighting circuit, and the battery does little or no work except when the power station is shut down. The principal advantage of this arrangement is that it is practically automatic in its action, and requires no regular attendant. The station employes can handle the switches when necessary to turn on or off the lights. An occasional inspection of the battery is all that is necessary. The cost of such lighting is much less than it can be purchased from lighting companies. In larger stations requiring several hundred lights, a motor generator can be used instead of the rheostat and connected to the battery and the load in exactly the same way. Such a plant should have an attendant.

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

NEW YORK, December 27, 1899.

The 138th Meeting of the INSTITUTE was held this date at 12 West 31st Street and was called to order by President Kennelly, at 8.15 P. M.

The Secretary announced that at the meeting of the Executive Committee in the afternoon, the following associate members were elected:

HARDY, CARL EARNEST,	Student, Cornell University, residence, 306 Huestis St., Ithaca, N. Y.	L. S. Randolph. Fred'k Bedell. Harris J. Ryan.
JAQUAYS, HOMER M.	Lecturer in Mechanical Engineering, McGill University, residence, 862 Sherbrooke St., Montreal, Quebec.	R. B. Owens. L. A. Herdt. C. H. Davis.
MISAKI, SEIZO,	Chief Engineer and Superintendent, Hanshin Elec. R. R. Co., Front Sannomiyo Station, Kobe, Japan.	W. E. Goldsborough Harold B. Smith. K. Iwadare.
MOODY, VIRGINIUS DANIEL,	Senior Student, Cornell University, residence. 215 Dryden Road, Ithaca, N. Y.	Edw. L. Nichols. Fred'k Bedell. Harris J. Ryan.
TAYLOR, JEREMY F.	Electrician, Detroit Copper Company, Morenci, Arizona.	C. F. Bancroft. Ernst Berg. W. C. Woodward.
WOLFF, FRANK A., JR.	Professor of Physics and Electrical Engineering, Corcoran Scientific School, Columbian University and in office, U. S. Standard Weights and Measures, Washington, D. C.	W. D. Weaver. J. E. Woodbridge. T. C. Martin.

Total 6.

The following associate members were transferred to membership:

HEWLETT, ERNEST HOLCOMBE Electrical Engineer in Chief Control, Rockhampton Gas and Coke Co. Ltd.; residence, Estoril, Rockhampton, Queensland, Australia.

KNOX, CHAS. EDWIN With C. O. Mailloux, Consulting Electrical Engineer, 150 Nassau St., New York.

BURTON, WM. CORWIN With J. G. White & Co., 29 Broadway, New York City.

THE PRESIDENT:—The first business before the INSTITUTE this evening is a paper upon the "Cost of Arc Lighting," copies of which you have before you. This is a communication in discussion of Prof. Robb's paper that was read here on September 27th. Mr. Wait is unfortunately not here this evening, but Mr. Albright has kindly consented to read the communication in his absence.

[See p. 579 November issue.]

THE PRESIDENT:—Accompanying this paper are sixteen diagrams and also a summary, to which the whole paper appears to tend, appearing under thirteen columns. The paper is now before you for discussion.

DISCUSSION IN NEW YORK.

MR. C. O. MAILLOUX:—I regret very much that I am not able to contribute to the discussion, because I have only just seen the paper and the diagrams. I am greatly impressed with the evident care and thoroughness bestowed upon it. The paper seems very valuable and it evidently contains a great deal of useful information in convenient and complete form. I regret very much that I have not had time to study it so as to be able to discuss it intelligently. The paper is not perhaps one that admits of, or needs much discussion. The paper is really a statement furnishing comparative data very useful for reference which the members of the INSTITUTE can have before them whenever they wish to investigate matters of this kind.

It affords perhaps a more complete presentation than any yet attempted of the points of difference and of the items of relative cost which should be considered in making a comparison between various methods of operating arc lamps.

Wait on Cost of Arc Lighting.

(See p. 579 November issue.)

Discussion in Chicago, December 27, 1899.

Mr. F. N. Boyer:—I had hoped to come prepared this evening with some figures based on the actual operation of the Hartford system in a number of stations in which it has been installed throughout the country. It was only a few days ago that my attention was called to the very interesting paper read by Mr. Wait this evening, and, owing to the holidays intervening, and some rather pressing business, I was unable to get what I wanted; and, therefore, rather than discuss the figures which are given by Mr. Wait, I will refer briefly to what has been done in the installation of the system.

Something over a year and a half ago the first installation was made at Hartford, Conn. Since that time something over 6,000-light capacity has been installed in other cities. In nearly every instance the installation was not made until after the Hartford system had been inspected, either by the prospective purchasers or by engineers appointed by them. Of course, in the first place, the only place to inspect it was at Hartford. Since then so many installations have been made in various parts of the country, that it was not necessary to go to Hartford; plants could be seen nearer home.

A notable instance of the installation of this system is at Omaha, with 400 lights used for street lighting, where the direct-current open type has been replaced by the same number of the alternating-current, enclosed series, 6.6-ampere lamps.

In St. Louis, where, you all know, the Missouri-Edison company has been practically operating the Hartford system, but instead of having an automatic regulating transformer, it has been using a transformer with fixed coils and a hand regulator, using the open arcs instead of the enclosed arcs. When the Missouri-Edison company installed its underground three-wire system instead of putting in the open arcs, it put in enclosed arcs, using, however, a multiple arc, instead of a series arc. The reason for this was that the company wanted to connect the lamps to its system of three-wire mains. The St. Louis people and the Omaha people both have large installations, and they, as well as the owners of a 100-light installation at Adrian, Mich., report very favorably. The general public, the people whom the lamp owners are trying to please, are as well satisfied as they were with the old system, and, in some cases, better.

It has been clearly established, of course, from a commercial point of view, that the enclosed arc lamp is a success for street lighting. As to its economy in operation, we don't claim that for a station operating street lighting alone—that is not doing any commercial lighting—the Hartford system is

the system to install. In that case, we believe that the direct-current, operating from a direct-current series machine, with enclosed arc lamp, is the system to install. For a local lighting company doing a commercial business as well as street lighting, there is no doubt at all that the Hartford system, used in connection with the alternating machine, is the proper system to install. Take Omaha, for instance. The company in that city some few years ago had a station, I think, of three stories, and each one of the stories was filled with operating machinery. Now there is a one-story building, since the installation of the Hartford system, and the company has scarcely filled the first floor, using the alternating current successfully, with large units, by which motors, arc lamps and incandescent lamps are supplied from the same source of power. That is where we claim that the alternating series system is economical.

I am sorry that the time was so short that we could not present to you some figures showing the actual operations of the Hartford system. At some future time I will, perhaps, be able to give you those figures, because the system has been watched very closely, both by the managers of the companies that have installed the system and by our own engineers, and we are now just about prepared to give figures based on actual operations.

MR. GEORGE A. DAMON:—Our office recently adopted a system of street lighting, for a suburban town, which allows a combination of one circuit of series alternating-current arc lamps controlled by an inductive regulator, and two or three circuits of series incandescent lamps, both arcs and incandescent lamps being run from one alternating-current machine.

Considering the best system to be adopted for these small towns, there are a number of things to take into consideration. The question of relative efficiency and total cost of installation and operation of arc-lighting systems has been very thoroughly covered in Mr. Wait's paper, but these items of cost and efficiency are not always the governing considerations—especially in plants of moderate size. In such a case it is desirable to locate arc lamps in the center of the city, but in the outlying districts, where the population is scattered, incandescent lamps are satisfactory. As has been pointed out, the mean illumination from incandescent lamps may be higher than that obtained from the same amount of energy used in arc lamps, but, on the other hand, there is no doubt but that a town lighted by arcs appears to be the better lighted. Most city councils like to have their towns look, from the railroad at least, as if they were well lighted, and the advantages of arc lighting for this purpose generally appeals to them. In the installation already referred to, there was considerable commercial incandescent lighting to be taken care of, besides the arc and incandescent street lighting, so that the best system was thought to be an alternating-current dynamo, capable of handling all classes of service, and the only question considered

was the relative advantages of the two or three commercial systems which would accomplish this result.

I noticed that Mr. Wait, in his discussion, has reduced all values, for the purpose of comparison to the cost per candle-power. This compares the quantity of light given off by an arc without taking into account the quality. Now, it has been often pointed out that there is considerable difference between the candle-power of an arc lamp and its illuminating power. Furthermore, the candle-power of an arc lamp is a hard thing to measure with any given lamp. When it comes to a comparison between all the different kinds of arc lamps—open arcs, enclosed arcs—both alternating and direct current—there is considerable variation in the reported results, and any deductions of relative cost based upon these figures will necessarily be as unsatisfactory as are these reported results themselves at the present time.

You will notice that Mr. Wait uses the ordinary rating of 2,000 candle-power for the open series-arc lamp. This rating has always been unsatisfactory, and it is unfortunate that we don't make some effort to get away from it. Allowing 2,000 candle-power for a 450-watt lamp, there will be only 0.225 watt per candle-power, whereas, in the values quoted from Professor Thomson and L. B. Marks, the watts per mean spherical candle-power for open arcs are given at 1.2 and 0.95, which indicates that these authorities are using candle-power ratings of about one-fifth the 2,000 candle-power value usually taken. Fortunately, however, the ratio between the costs as given, and also the conclusions drawn would not change, even if the candle-power basis were to be taken at nearer its true value.

In the final table, showing the conclusions to be drawn from Mr. Wait's calculations, I was interested in noticing, in the comparison of the operating expenses per candlepower, that the cost is less (the value being 0.0313) for the station using the 450-watt series-arc lamp operated from direct-connected arc dynamos. This brings us back to the opening sentence of Mr. Wait's paper, in which he states that the figures were originally made to see whether or not it would pay to build a direct-current arc machine suitable for direct connection. I would say that his conclusion might be that it would pay, although he has not said so, and I should like to hear his conclusion on that subject. It strikes me that the great want at the present time in power-station work is a direct-connected arc dynamo—a machine we can install in our stations on the same basis as the large alternators; that is, a machine that can be built in large sizes and directly connected to a Corliss engine without a countershaft.

Such a plant is shown[1] by Fig. 9, and if I read Mr. Wait's figures aright, this arrangement shows the best results from an operating standpoint. The diagram shown in Fig. 9 provides for a direct-

1 *Western Electrician*, January 13, 1900, page 25.

connected arc plant, and also for other units to be used for other purposes. Owing to the fact that the arc-lighting load and the commercial incandescent and power load will overlap under ordinary circumstances, it will usually be found desirable to have separate dynamos for the arc-lighting circuits, and the arrangement suggested indicates a desirable one for many stations. For instance, the center engine could be a center-crank engine, directly connected to two large arc dynamos. The unmarked units at each side could be alternating-current, or even direct-current generators for commercial work, directly connected to side-crank engines. It is hardly necessary for me to point out that this arrangement would be much more satisfactory and reliable if all the engines and generators were interconnected by means of the Arnold system, so that the crippling of any one engine unit would not interfere with the output of the plant. One of the most interesting points brought out by the paper is whether or not we are tending toward a large direct-driven arc machine.

MR. ALEXANDER CHURCHWARD:—I should like to ask Mr. Boyer one question in regard to the power factor of that system (if he were here), because I think, from what I understand of the constant current system of the General Electric Company, that you get a comparatively low power factor, and if very low, of course, it will upset the regulation of your machine. I understand that the power factor is from 60 to 70 per cent. under full load. Now, if you have a power factor of 70 per cent., a much larger generator will be needed to supply a given number of lamps than if the arc machine were driven by a direct-current or synchronous alternating-current motor. Our losses with the low-power factor would increase and cut down the efficiency of the generator, and our operating expense certainly would go up, and also the cost per lamp; because with the low-power factor we need a larger machine than with a high-power factor. That would increase the cost of the alternating lamp per candle-power and spoil the regulation of the system. Therefore, if we are going to spoil the regulation by putting on a series-alternating lamp, that again will be a loss. For instance, if you take a small plant of 200 kilowatts and put on lamps equal to 50 kilowatts with 70 per cent. power factor, that is going to interfere with the regulation of the rest of the load.

MR. FOSTER:—We do it at better than 70 per cent.

MR CHURCHWARD:—Well, what is it?

MR. FOSTER:—You can search me.

MR. CHURCHWARD:—If it is any better than that let us know what it is.

MR. WARNER:—I can answer that. I was assured by Mr. Sunny the other day that the system gives a power factor of a little over 80 per cent. under full load.

MR. CHURCHWARD:—What would it be under three-quarters load?

MR. WARNER :—I didn't get that.

MR. CHURCHWARD :—Taking any of our existing stations to-day, where we want to run arc lamps, if the machine is composite wound and the voltage rises 10 per cent. under full load, with a 95 per cent. power factor, if the power factor is reduced down to from 80 to 75 per cent., we lose our regulation. If we are going to lose our regulation by using that system, I don't think we would gain much by putting in a synchronous motor and driving the arc machine. So that the power factor being better, we would then get better results and better efficiency, and the cost per lamp would be brought down, as shown by the diagrams; we would not upset the regulation of our station, and I think this is one of the main points It is poor enough now. I don't think it advisable to put in a system that is going to reduce the regulation 10 or 15 per cent. unless we have a man to watch it all the time, which means an additional expense.

MR. E. P. WARNER.—I am sorry to say I have been so busy lately that I have had little time to prepare myself for the discussion of this subject this evening. I have had occasion to look over a number of statements that have been made regarding the power factor of the various alternating systems of arc lighting, and among them, of course, the Hartford system. I have noticed by some advertisements that the power factor was as low as 70 per cent. at full load. I asked people who knew what it was, and was assured that it was about 80 per cent. I feel just as Mr. Churchward does, in regard to the matter of the effect on generators. Unless we can get a series alternating system in which the power factor is certainly not less than 90 per cent. we can't be considered in a position to compete with the direct-current, enclosed-arc system.

Coming to the tables that Mr. Wait has prepared, they show very thoroughly and fully some very interesting points regarding the comparative cost of operating plants. I don't find anything there, however, that would approximate the conditions of a plant such as Mr. Boyer has called attention to. That is, one in which a large amount of the business was "commercial" lighting, using ordinary incandescent lighting, either commercially or on the street, and in which only a very small proportion of the business is arc lighting.

I should be very much interested, if there is anyone present who can give it, to have information as to the real economy that can be obtained by the use of the Hartford system, or some system akin to that, in, say, an alternating station that has been already established, where they are doing constant-potential arc lighting and incandescent lighting, and power work.

MR. R. H. PIERCE :—I would like to call on Mr. Wait again in regard to this matter brought up by Mr. Damon; that is to say, as to his conclusion on the advisability of installing large direct-connected, direct-current units, and whether or not there

is a tendency in that direction. One thing is certain, that the economy which various companies are trying to get by these different systems, they aim to bring about by the use of large units, which, perhaps, in general, will increase the efficiency and decrease the labor account, which is a considerable item. And I suppose that any analysis would show that a large proportion of the decrease in expense in any system is due to the use of a few large units. Therefore, it is very pertinent, I think, for us to learn the status of the large connected direct-current machine.

MR. WAIT:—Taking up some of the points that have been mentioned in the discussion, I might first call attention to Mr. Boyer's remarks that the Hartford system has been installed in a very large number of plants, and that it has almost universally given satisfaction. This really comes back to the same point that is taken up at the very beginning of the paper—that the Hartford system, in practically all cases, I think, has displaced some of the older forms of open-arc systems. It is more than likely that in a great many of these cases a modern direct-current system could have displaced the older open-arc systems and have furnished better light to the public at a less cost to the station.

In this connection I have heard recently of a town near here, where they are operating both the alternating and the direct-current systems and the old direct-current company is making life miserable for the alternating company by poking the public up to perceive that the alternating lamps don't give out quite as much light as their lamps.

The alternating systems and the enclosed direct-current arc systems offer a very fine opportunity of putting up a confidence game on the public. That is to say, it will quite frequently be possible to convince city councils or other uninitiated persons that a lamp taking less watts will give more satisfaction for street lighting than some of the old open lamps. That this is possible will readily be seen from a comparison of the candle-power curves of open and enclosed lamps, either direct or alternating. The enclosed lamps, especially the direct current, give considerable more light near the horizontal than the open arc, so that actual photometric measurements, taken in the middle of a long block, from an open lamp at one end and an enclosed lamp at the other end, will show to the advantage of the enclosed lamp. The advantage of the enclosed lamp is, perhaps, even more apparent to the eye, because the effect of the comparatively diffused light from the enclosing globe seems to make a greater impression than the twinkling beam of the open arc. Quite frequently, the observer will think that the enclosed lamp is the brighter, judging by looking at the lamp, while, if he looks down at his shadow, he will see that the open lamp is making the darker shadow.

Another point that influences such tests, is the extreme varia-

tion in the light from an open arc; that is, if the carbons are arranged either by accident or design so that there is considerable shading of the crater, the light in the direction of observation may be quite small. The enclosed lamps, of course, are not subject to nearly so great variations in this respect, partially on account of the greater length of the arc, and partially on account of the superiority of the more recent mechanisms. For example, a 1,200-candle-power open arc, with carbons poorly adapted to the conditions, and having the ends overlapping, will give practically no light in certain directions.

The light on the sidewalk near the open lamp is, of course, much greater than from the enclosed lamps, but in a great many cases it is possible to convince purchasers that this very brilliancy is a disadvantage, because it makes the distant regions seem even darker, on account of the comparison. In addition to the other arguments in favor of the enclosed lamp, its superior steadiness and the greater reliability of the newer mechanisms make it find favor, both in the eyes of the public and the station man.

Most of us would probably be glad to see the old systems displaced by enclosed lamps, but it is only fair play to the open arcs to bring out some of these points and show that a great many of the evils can be cured by the use of modern mechanisms, carbons adapted to the conditions, and globes which will diffuse the light or distribute it in the direction desired. The city of Brooklyn might be cited as an example, where the authorities would not permit the substitution of alternating enclosed lamps for direct-current open lamps.

Mr. Boyer has referred to the cost of operation and other matters in connection with the Omaha Thomson-Houston company. I have a letter from Mr. White, of this plant, in which he gives an account of some photometric tests he made out in the street, in which he obtained more light from the alternating 450-watt arc than from his open direct-current 450-watt arc. The results of these observations are evidently what might be expected, and the difference between these results and those given by Mr. Steel in his communication is probably due to the angles of observation and other differences, a number of which have been previously mentioned.

Coming to the plant mentioned by Mr. Damon, I would like to ask how many arc lamps there were and what system was adopted?

Mr. Damon:—There were 17 lamps to be installed on the Manhattan system.

Mr. Wait:—It is evident that the small number of arc lamps is alone a sufficient reason for not considering direct-current arc machines, and that this fact also puts the Hartford system at a disadvantage commercially. This question of the ratio of the arc-lighting load to the rest of the plant is the all-important one in deciding between the direct-current and alternating arcs. It

will be seen that the reasons for deciding on alternating arcs in this plant will be maintained in considerably larger plants, where the arc load is still a very small portion of the total.

Mr. Damon and Mr. Pierce have both asked about the possibility of obtaining large arc machines. I might mention that our company (Western Electric Company) has sufficient confidence in the future of large direct-current arc machines to undertake the development of a line of multi-circuit machines. The larger sizes of these machines are not ready for the market as yet. There has also been some mention in recent periodicals of the development of large arc machines by other companies.

Mr. Churchward has asked about the power factor of the alternating system at light load. The power factor at three-fourths load in the Hartford plant was stated by Professor Robb to be 62 per cent and 24 per cent, at one-fourth load.

This question of power factor brings up a point in connection with the operation of the plant. Suppose, for example, we have an alternating plant in which the same generators and engines are to be used, both for the arc-lighting load and other purposes. If the engine has the same capacity as the generator, it will not run at an economical load when used on an arc load of low power factor, unless the generator is run at a considerable overload. It is for this reason that in the tables, the generating units for the arc load have been arranged in proportion to the power factors, so that the generator and engine would reach their economical limit at about the same time. If one of these units is used on a nearly non-inductive load, the generator will then be working at less than its maximum economy. Where all generators can be run in parallel and the arc lighting is a small part of the whole, a low power factor in the arc load will, of course, not have so much influence.

There has not been very much said about alternating inductive regulator systems. I am sorry there is nobody here to speak up for these systems, as they undoubtedly have a good many advantages that might be brought out. I have no figures on the actual results obtained in practice. In the published accounts of some of these systems, power factors as high as 90 per cent are claimed, and from the results of some tests that I have in mind, I think it is quite possible to obtain results very nearly as good as this, and even better in some cases. This very feature of a high power factor gives such a system a considerable advantage over other alternating systems. An unusually high efficiency is also claimed for some of the lamps used on these systems.

Mr. Warner has brought up the comparison of alternating and direct-current systems for "commercial" lighting. It is evident in general, that the alternating systems would have somewhat more advantage for this class of work than would be indicated in the tables. The motor-driven or rectifier systems, however, share these advantages with the straight alternating systems, and

the latter have the great disadvantage mentioned in the paper, of either not operating satisfactorily at all on light loads or having quite low power factors. Tables 10 and 11 are intended to be especially applicable to these conditions.

Mr. Pierce spoke of the increased economy of large generating units on account of less attendance, etc. I might mention here an arc plant in which there are ten 150-light machines and two engines, in which there are only two men in the engine room. There is also another plant in which there are 12 machines, a countershaft and three engines, in which there are only three attendants in the engine room. These plants are both three or four years old and have two well-known makes of arc machines. From a consideration of these plants it is difficult to see how there can be any great difference in the item of attendance between modern arc machinery and other larger generators. It is quite evident that there will be some difference, but the item is so small that it does not affect the total cost very materially.

Before I stop, I would like to emphasize the desirability of getting some relative nominal rating for the different kinds of arc lamps, otherwise both parties to a contract will have all sorts of difficulty in settling on a reasonable basis. It does not seem possible to adopt any actual candle-power ratings, without giving rise to endless controversy. The hopelessness of rating by actual candle power is emphasized by the suit now pending in a good-sized city, where all sorts of contentions are made concerning the light of a standard form of open-arc lamp. If there can be so much controversy over a single type of lamp, which has been in use for so many years, what may we expect when we have competition between all the various forms of lamps now on the market. I understand that some steps toward standardization have been taken by the National Electric Light Association. It would seem very desirable to discuss this matter as fully as possible, so that the association or some other representative body can get together all the information there is on the subject.

MR. F. L. MERRILL:—Being only a privileged guest, this evening, I have hesitated to say anything until invited. I can only say that our company has the Manhattan system, which includes the reactive regulator and the new-type concentric lamp, the lamps being without series coils and having only one magnet therein. We have a large amount of business that calls for the use of no transformers. This system permits the use of a series of about 25 or 26 lamps connected directly across 2,000-volt primaries, which are being installed all over the country. In case people want to use a higher number of lamps than 25, a step-up transformer is a very simple proposition. In a large installation, an alternating unit of the required voltage, operating nothing but arc lamps, may be used. An example of this is the Los Angeles and San Gabriel Electric company, at Los Angeles, Cal., having nine circuits of 100 lamps each. These will be oper-

ated from a 7,500-volt generator without transformers. We are installing at the Union Stock Yards, for Swift & Co., 150 lamps, which will burn in six circuits, two circuits on a transformer, and each transformer on one phase of a three-phase system. In this case we shall step up from 220 volts to 2,000.

Regarding lighting from alternating lamps, my own impression, from observation, is that the light is more satisfactory and more reliable for street lighting. The average grade of carbon that is used in open arcs is not very satisfactory, and much depends upon that as to the degree and amount of light. The open-arc, direct-current lamp is complicated and throws shadows directly under the lamp, and a decrease in efficiency is noticed in a very short time. The enclosed lamp, of course, is not affected by the wind. In the new type of lamps which we are putting out, the side rods are only about one eighth of an inch in diameter, and throw practically no shadow at all.

In my own observation, near my residence, there is a block lighted, by private subscription, from the mains of the Commonwealth Electric Company, and on the next street, a parallel boulevard block, that is of the same length, on which the high-tension system is in use. There are more lamps on the boulevard, all of the latest type, with opal globes, and I believe that the first block, with a smaller number of six-ampere alternating lamps, is better illuminated.

The Manhattan system has been put in in a great many places, replacing small plants of open arcs. The satisfaction is almost universal. The diffusion of the light from the globes of the enclosed arcs seems to me more satisfactory for general illumination.

I believe that the alternating system, compared with the old-style, direct-current arc system, which it will probably supplant, in a great many cases, is a very fair test of the efficiency of that system. It represents a dismantling and throwing out of almost obsolete types of machinery in a great many cases. It means the throwing out of old types of lamps that probably have not been kept in repair. The average distribution of light from a direct-current arc is not the best. Most of the six-ampere lamps are better than the direct-current lamps of ten amperes.

Putting in the alternating-series system, under ordinary circumstances, is to place it in the hands of inexperienced men. I think if the system will operate satisfactory under those conditions, that it is a very fair test. There is no question about the efficiency of an alternating system, with the series lamps under our system. We are prepared to guarantee a power factor of 89 per cent. at full load. As to partial loads, I cannot say; but I promise it will not run under the Hartford system. The loss in our regulator is constant, being simply the I^2R loss and a small iron loss. The loss in a 50-light regulator is only 150 watts. In fact, in a 50-light system the loss is equivalent to one lamp,

as against five lamps in the Hartford system. The regulator has only one coil, and can be replaced, in case of a burn-out, in 10 minutes. There is absolutely nothing else to get out of order. The company has sold and put in nearly 3,000 lamps in the last six months.

MR. WAIT:—I would like to ask Mr. Merrill about the San Gabriel plant that he has mentioned. There were nine circuits of 100 lamps, I believe, at 7,200 volts' constant potential.

MR. MERRILL:—It is about 7,500.

MR. WAIT:—I would also like to ask what provision is made for taking care of a ground on the positive side of a circuit when there is one on the negative side of one of the other circuits. If the regulators were to take care of only a portion of the circuits, one would expect to see the whole system shut down, unless it was protected by other automatic devices besides the regulator.

MR. MERRILL:—I do not believe I am sufficiently familiar with that installation to reply exactly. It is, however, provided with regulators which can be made to take care of a large part of the capacity of the circuit. A ground coming on the opposite sides of any two circuits of the system will, so far as I can say offhand, cut out the number of lamps that the regulator would take care of in the portion grounded.

MR. WAIT:—It would seem that in order to get complete independence of a large number of circuits on such a system, it would be desirable to have static transformers in between the generators and the series system. That is the reason that wherever such systems are shown in the tables, an addition has been made for static transformers. Since making up these tables, it has occurred to me that it would not be necessary to have transformers for the entire capacity; but that a certain number of transformers might be provided to throw in where trouble occurred.

Mr. Merrill has also spoken of the direct-current lamps being somewhat complicated. I suppose that is a matter of opinion; but there is one thing certain, and that is that the alternating lamps are liable to produce a humming, even under the best conditions.

I think there is no doubt that an alternating-lamp mechanism is intrinsically a more difficult piece of apparatus to maintain than a direct-current mechanism of the same character. In actual practice it has been found quite difficult to maintain alternating lamps in service with anything like the amount of attention given to direct-current lamps. It is only fair to the alternating lamps to state that there has been quite a rapid improvement in the lamps that have appeared recently.

Another point to which Mr. Damon has called attention is the intermittent effect of the light on moving objects, from an alternating arc of low frequency; that is, a moving object will be seen as a series of impressions, instead of in the ordinary way. This might be somewhat annoying, under certain circumstances.

I understand that the light from an incandescent lamp run from a very low frequency current sometimes becomes quite painful to the eyes, even though the person does not realize the fact that the light is varying. It is my impression that where such cases have occurred, the trouble has been overcome by using a low-efficiency lamp. One would expect that the same effect would occur to a more marked degree from the lower frequency alternating lamps, if these were situated where they were used for work requiring close application.

Mr. Merrill spoke of the power factor of the inductive regulator system as 89 per cent. I would like to ask the size of regulator with which this figure corresponds, that is, if it is a 100 per cent. regulator, what will be the power factor of a circuit with a 10 per cent. regulator.

MR. MERRILL:—We manufacture a number of sizes of regulators adapted to different uses. For instance, a 100 per cent. regulator for 25 lamps would be a 50 per cent. regulator for 50 lamps, and so on in that proportion. The loss in the regulator being constant at all loads, we can maintain that power factor. In actual tests, we have obtained 91 per cent.

MR. WAIT:—It would appear, then, that the size of the regulator has some influence on the power factor?

MR. MERRILL:—That is, the capacity of the regulator.

MR. WAIT:—Yes, that is, the per cent. of the circuit it can take care of.

MR. MERRILL:—Our statement is based on one hundred per cent.

MR. WAIT:—Another statement made was that the loss in the regulator was constant. We would naturally suppose that the hysteresis loss might vary somewhat with different loads.

MR. MERRILL:—I believe I made the statement it was an I^2R loss with a small iron loss.

MR. WAIT:—Which, of course, would vary a little.

MR. MERRILL:—Yes.

MR. WAIT:—Do you know how much that iron loss is?

MR. MERRILL:—I do not.

Mr. GEORGE B. FOSTER:—Mr. Wait, did I understand you to say that you had figured the price of transformers in your proposition number ten?

Mr. WAIT:—Yes, the transformers are figured in that; also in this one here (indicating.) This is practically the same thing as the Manhattan system.

MR. FOSTER:—I do not see why a transformer in that system would be necessary, as you already have the voltage required on the primaries. Regarding short-circuit on the line, suggested by Mr. Wait, this will be taken care of by the regulator, which would quickly choke the circuit, preventing any more serious damage than occurs on the direct system, when operated by an automatic series dynamo.

MR. WAIT:—I think that in actual practice, where there is a considerable amount of alternating series lighting in conjunction with a constant-potential alternating system, it will be found desirable to separate the series system by static transformers or some other method, on account of the quite frequent occurrence of grounds of both high and low resistance on series arc circuits. These grounds, while they may or may not shut down parts of the system, would be a source of considerable annoyance, and even danger, to other parts of the system. As mentioned in the description of the tables, the static transformers were put in partially on this account and partially from the fact that, as a general thing, the usual unit voltage of series arc circuits is higher than the primary voltages commonly used in stations, so that step-up transformers would quite frequently be required.

MR. FOSTER:—In a case of that kind, one large unit would be all that would be necessary, and the various circuits could be taken from the secondary or higher pressure side, each circuit to be governed by one of the regulators.

MR. WAIT:—Yes, if you wanted to do that, it would bring the transformer cost down.

MR. FOSTER:—We are considering the case particularly of a station doing arc lighting exclusively. This is the condition most favorable to a direct current system.

MR. WAIT:—In the conclusion, however, it is stated that if the plant is for an arc lighting business alone, the conditions are more favorable to the direct current plants. The figures for such an arc plant will not vary sufficiently from those in the tables to make it worth while to get up another set.

MR. FOSTER:—Mr. Boyer and Mr. Merrill have cited instances of large central stations operating a series of alternating arc lamps. The station Mr. Boyer spoke of, at St. Louis, is furnishing in the neighborhood, I believe, of 3,000 lights, with power and incandescent lighting from the same generator for the series arc lighting, with minutely divided secondary coils. The regulation is accomplished by cutting in or out these coils to keep the current constant. This regulation is done at present by hand. There is no reason, however, why this could not be accomplished automatically. The power factor in a system of this nature would be practically constant at all loads, depending upon the arc lamps used. With the best Manhattan lamps, I am advised that they get a power factor of 90 per cent. Therefore, with an automatic regulator for the transformer, and a good lamp, you could maintain a high-power factor at all loads, and have an excellent system in every particular. With this regulation at the transformer, you will understand there will be no necessity for a regulator similar to the Manhattan or General Electric type.

They are using transformers in St. Louis similar to these I have just described, without the automatic regulation. These transformers were installed and perfected, I believe, previous to the introduction of the Hartford and Manhattan systems.

MR. WAIT:—You will notice that such a system is somewhat in the nature of a cross between the inductive regulator and constant-current transformer systems shown in the tables; that is to say, there is a transformer, which, by the varying relation of primary and secondary, gives the range of voltage desired, while the current is maintained constant under steady conditions by the inductive resistance of the circuit.

It is seen that both the constant-current transformer systems and the inductive-regulator systems show very good economy, as far as everything but the arc itself is concerned. The whole question is practically between the direct and alternating arcs. It is quite evident that a station arrangement like that in St. Louis has a great many advantages over any plant where the dynamo and engine units have to be subdivided into a great many more types and sizes. The question is not one of comparing stations, but of entire systems, including the arc itself, and the contention is made that for a great many plants the superior efficiency of the direct-current arcs will more than make up for the corresponding loss in the station.

I have had some reports on the relative values of the lamps in St. Louis, but they vary so much that it is not worth while to mention them. That is, you find some people who will say that the alternating lamps give very much better light than the direct-current. Another man claims to have made photometic tests from which he found that the alternating lamps were not giving half the power that the direct-current lamps were.

MR. FOSTER:—I don't mean to get into a discussion of the relative merits of the different lamps, but as to the comparative systems applicable to the trade. It is a fact, I believe, that the great majority of plants, taking small and large together, are operating alternating machines. It is a fact that a plant can be installed with series arc lamps so that the power factor will be practically constant, and a very high power factor at that. And the lamps, as Mr. Merrill says, are quite as simple as the direct-current lamps.

Taking that into consideration, it seems to me that purely from the commercial side of the question, it is a better general proposition than the installation of separate arc machines for the series lighting with an alternating plant for the incandescent lighting. I don't believe that anyone wants to question the statement that a series system, either alternating or direct, is preferable to the multiple system; that is, putting the regular lamps in multiple on 110 volts.

MR. WAIT:—There is no doubt, whatever, that a station having all alternating machinery is better than a mixed one, but the relative amount of light from the arcs should be taken into consideration.

[Adjourned.]

A paper presented at the 130th Meeting of the American Institute of Electrical Engineers, New York, January 24, 1900. President Kennelly in the Chair.

NOTES ON SINGLE-PHASE INDUCTION MOTORS,

AND

THE SELF-STARTING CONDENSER MOTOR.

BY CHARLES PROTEUS STEINMETZ.

I.—GENERAL.

1. In two previous papers, July, 1897, and February, 1898, of the TRANSACTIONS, I have given the theory and calculation of the polyphase induction motor and the single-phase induction motor, and shown that at or near synchronism the magnetic field in either can be considered as consisting of two equal magnetic fluxes, superposed upon each other in quadrature in time and in space.

In the polyphase motor both of these fluxes are proportional to the impressed E. M. F. less the impedance voltage of the primary current (the latter considered in its proper phase relation) thus equal to each other at all speeds. In the single-phase motor only one of the magnetic fluxes, that in the line of magnetization of the primary exciting coil, or the main magnetic flux, is proportional to the impressed E. M. F. less impedance drop, while the second magnetic flux, in quadrature with the axis of the primary exciting coil, or quadrature magnetic flux, falls off from equality with the main magnetic flux at synchronism to zero at standstill, or if by a starting device a quadrature E. M. F. is impressed upon the motor in starting, to the quadrature magnetic flux given by the starting device, or auxiliary magnetic flux. In consequence thereof, while in the polyphase motor the torque equals secondary energy current times induced E. M. F., $e^2 a_1$, in the single-phase induction motor it is secondary energy current times quadrature induced E. M. F., $e^2 a_1 (1-s)$, when as-

suming the quadrature E. M. F., and thus the quadrature flux to fall off proportional to the slip s.

2. Thus the calculation of the single-phase motor load curves is the same as that of the polyphase induction motor, except that:

1st. In the single-phase induction motor in the equation of torque, the factor $(1-s)$ has to be added.

2nd. In the single-phase motor the admittance and the impedances are different from those of the same motor on polyphase circuit.

Since the magnetic field of the single-phase and the polyphase motor is the same at synchronism, the total volt-amperes excitation are the same, that is in the three-phase motor on single-phase circuit the total exciting admittance is three times, in a two-phase motor on single-phase circuit it is two times that of the same motor on polyphase circuit.

The primary self-inductive impedance on single-phase circuit is the impedance of the circuit or circuits used.

The secondary self-inductive impedance is the joint impedance of all the secondary circuits, that is one-third the impedance per secondary circuit with three-phase, or one-half with two-phase wound secondary or armature.

3rd. In the single-phase motor the characteristic constant: $\vartheta = 2yz$ is twice that of the same motor on polyphase circuit and the behavior of the motor thus correspondingly poorer.

3. While the load curve of the single-phase motor calculated in this way gives a fair agreement with the test at or near synchronism, at intermediate and low speeds, especially with motors of high magnetizing current, as high-frequency motors, experience shows that a more accurate investigation of the internal reactions of the motor is necessary; and in view of the rapid introduction of self-starting single-phase induction motors, in which the range of torque curve from standstill upward is of importance, the results of this investigation will be given in the following:

Primary Admittance.—At synchronism the armature or secondary of the single-phase induction motor is not without current, but carries the exciting current of the quadrature magnetic flux. The primary thus carries a current equal to the secondary exciting current or exciting current of the quadrature flux (reduced to the primary by the ratio of turns) plus the ex-

citing current of the main magnetic flux. Thus if Y_0^1 = exciting admittance of the main magnetic flux, at synchronism, where main flux and quadrature flux are equal, the secondary exciting admittance or admittance of the exciting current in the secondary of the motor is $Y_1^1 = Y_0^1$, the total exciting admittance thus $Y^1 = Y_0^1 + Y_1^1 = Y_0^1$.

At standstill in the absence of a starting device, the quadrature flux and thus the secondary exciting current have disappeared. If a starting device is used which impresses upon the motor at standstill an auxiliary quadrature flux $\Phi_1 = t\,\Phi_0$, where Φ_0 = main magnetic flux, the secondary or quadrature exciting current at standstill corresponds to Φ_1, that is, is t times the exciting current of the main magnetic flux, and the total exciting current thus $(1+t)$ times that of the main magnetic flux.

Thus the two magnetic fluxes can either be considered separately, the main magnetic flux corresponding to a counter E. M. F. e equal impressed E. M. F. less impedance voltage (in its proper phase relation) giving the primary exciting current eY_0^1, and the quadrature magnetic flux corresponding to a counter E. M. F. e^1, varying from e at synchronism to te at standstill, and giving the secondary exciting current $e^1 Y_0^1$, varying from eY_0^1 at synchronism to $et\,Y_0^1$ at standstill; or both magnetic fluxes can be considered together as produced by the counter E. M. F., e equal to the impressed E. M. F. less impedance drop, but the main magnetic flux as having a constant admittance Y_0^1, and the quadrature magnetic flux an admittance varying from Y_0^1, at synchronism to tY_0^1 at standstill (or 0, if at standstill no quadrature magnetic flux exists). The latter method is the more convenient and thus preferred in the following:

The function, in which the secondary exciting admittance varies between standstill and synchronism, depends upon the nature of the starting device, and is in the absence of a starting device and with the monocyclic starting device and the condenser in the tertiary circuit, an elliptic function of slip, but can as first approximation be assumed as proportional to the slip, that is as linear function. The secondary exciting admittance at slip s is then: $Y_1^1 = Y_0^1\,[1-(1-t)s]$ in a motor whose starting device produces at standstill a ratio of fluxes t. The total exciting admittance at slip s is thus: $Y^1 = Y_0^1 + Y_1^1 = Y_0^1\,(2-s)$ or $Y^1 = Y_0^1 + Y_1^1 = Y_0^1\,Y_0^1\,[2-(1-t)s]$ respectively.

4. *Self-Inductive Impedance.*—At synchronism the secondary impedance of the single-phase motor is the joint impedance of all secondary circuits, that is one-third the impedance per circuit with a three phase, one-half the impedance per circuit with a two-phase armature, since all secondary circuits uniformly cor-

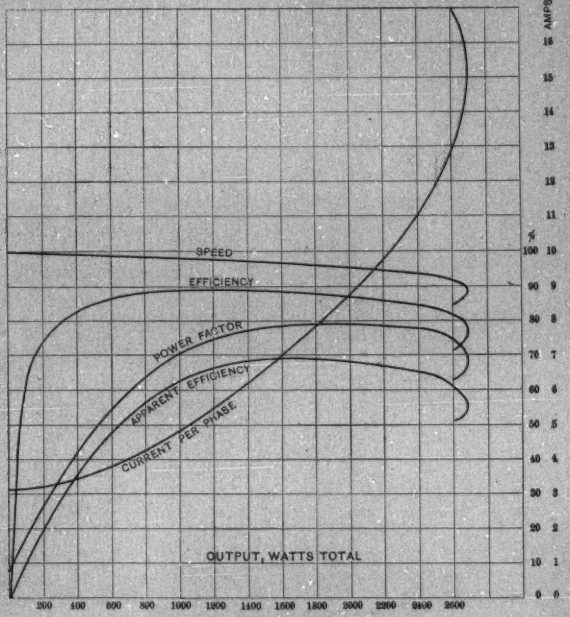

FIG. 1. Load Curves of Three-Phase Motor.

$$e_0 = 110.$$
$$Y_0 = .002 + .03\,j.$$
$$Z_0 = .6 - 2.4\,j.$$
$$Z_1 = .6 - 2.4\,sj.$$

respond to the primary circuit. At standstill, however, the secondary circuits correspond to the primary circuit only with their projection upon the plane perpendicular to the main magnetic flux, that is as far as interlinked with the primary flux. That is, the resultant secondary impedance at standstill equals

one-half the joint impedance of all secondary circuits. Thus from standstill to synchronism the secondary impedance of the single-phase motor with a three-phase secondary of impedance Z_1 per circuit varies from $\frac{2}{3} Z_1$ at standstill to $\frac{1}{3} Z_1$ at synchron-

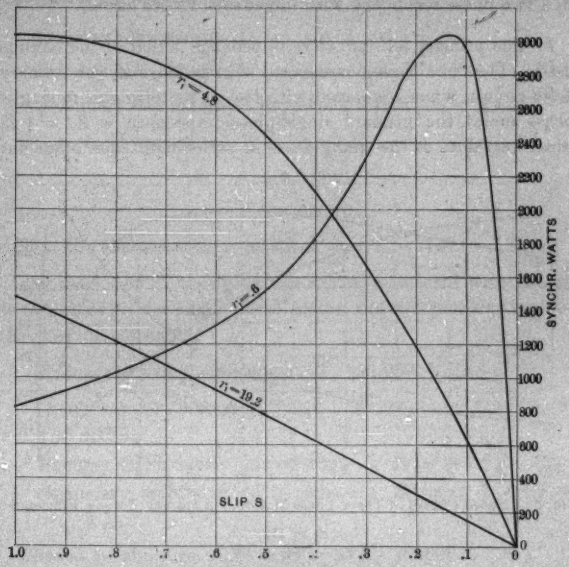

FIG. 2. Load Curves of Three-Phase Motor.

Torque.

$e_0 = 110.$

$Y_0 = .002 + .03\,j.$

$Z_0 = .6 - 2.4\,j.$

$Z_1 = .6 - \left.\begin{matrix} \\ 4.8 - \\ 19.2 - \end{matrix}\right\} 2.4\,sj.$

ism. With two-phase secondary of impedance Z_1 per circuit it varies from Z_1 at standstill to $\frac{1}{2} Z_1$ at synchronism, and as first approximation the resultant secondary impedance of the single-phase motor can be expressed as function of the slip s:

$$Z_1' = Z_1 \frac{1 + s}{3}$$

in a three-phase secondary with Z_1 per circuit.

$$Z_1^1 = Z_1 \frac{1+s}{2}$$

in a two-phase secondary with impedance Z_1 per circuit.

5. The primary self-inductive impedance is that of the circuits used. Thus in a three-phase motor of impedance Z_0 per primary delta circuit, when connected with two of its terminals to single-phase mains, the primary single-phase impedance is $Z_0^1 = \frac{3}{2} Z_0$ (since two sides of the triangle are in series with each other but

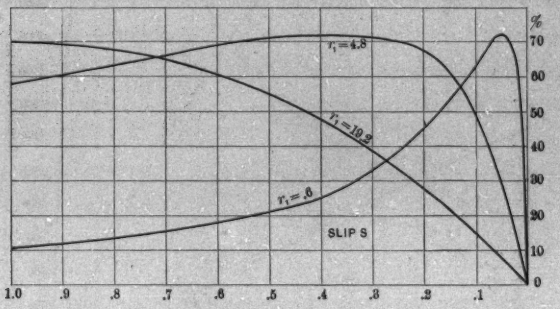

FIG. 3. Speed Curves of Three-Phase Motor. Apparent Torque Efficiency.

$$\begin{aligned} e_0 &= 110. \\ Y_0 &= .002 + .008\,j. \\ Z_0 &= .6 + 2.4\,j. \\ Z_1 &= \left.\begin{matrix} .6 - \\ 4.8 - \\ 19.2 - \end{matrix}\right\} 2.4\,j. \end{aligned}$$

in shunt with the third side). In a two-phase motor of impedance Z_0 per circuit, with one circuit used on single-phase mains, the primary single-phase impedance is Z_0.

The characteristic constant of the single-phase induction motor, or the product of exciting admittance at synchronism times total impedance at standstill, or approximately of exciting current over starting current, or of useful flux over waste flux, is twice that of the same motor on polyphase circuit, since in the three-phase motor the admittance is trebled, the impedance reduced to two-thirds; in the two-phase motor the admittance is doubled

and the impedances are the same on single-phase as on polyphase circuits.

6. As instances are shown in Fig. 1 the load curves and in Figs. 2 and 3 the speed curves of a three-phase motor of the constants per delta circuit.

Exciting admittance: $Y_0 = .002 + .03\,j$.

Primary self-inductive impedance: $Z_0 = .6 - 2.4\,j$.

Secondary self-inductive impedance: $Z_1 = .6 - 2.4\,j$.

Thus: Total impedance: $Z = Z_0 + Z_1 = 1.2 - 4.8\,j$.

$$Z = 4.948$$

$$Y_0 = .03007.$$

Thus the characteristic constant $\vartheta = y_0\, z = .149$.

The same motor on single-phase motor, connected with two of its terminals to the single-phase mains, and without starting device, has the constants:

Primary exciting admittance: $Y_0^1 = .003 + .045\,j$.

Secondary exciting admittance: $Y_1^1 = (1 - s)\,(.003 + .045\,j.)$

Primary self-inductive impedance: $Z_0^1 = .4 - 1.6\,j$.

Secondary self-inductive impedance: $Z_1^1 = (1 + s)\,(.2 - .8\,j.)$

Thus at synchronism: $Y^1 = Y_0^1 + Y_1^1 = .006 + .09\,j$.

$$Y^1 = .0902.$$

At standstill: $Z^1 = Z_0^1 + Z_1^1 = .8 - 3.2\,j$.

$$Z^1 = 3.30.$$

Thus the characteristic constant $\vartheta = y^1\, z^1 = .298$.

The torque is $T = e^2\, a_1\, (1 - s)$.

This motor fairly represents what can be done in a 1 K.W. 125-cycle single-phase motor. Its load curves are shown in Fig. 4 and the speed curves in Fig. 5. These curves are calculated in the way shown in my previous paper, with Y^1 as exciting admittance and Z_0^1 and Z_1^1 as self-inductive impedances, except that these quantities here are functions of the slip s, and not constants as assumed in my previous paper.

II.—Condenser in Tertiary Circuit.

7. The efficiency of the single-phase induction motor, while inherently lower than that of the same motor on polyphase circuit, is with better motors still quite fair. Very low, however, especially at light loads is the power factor, and thus the ap-

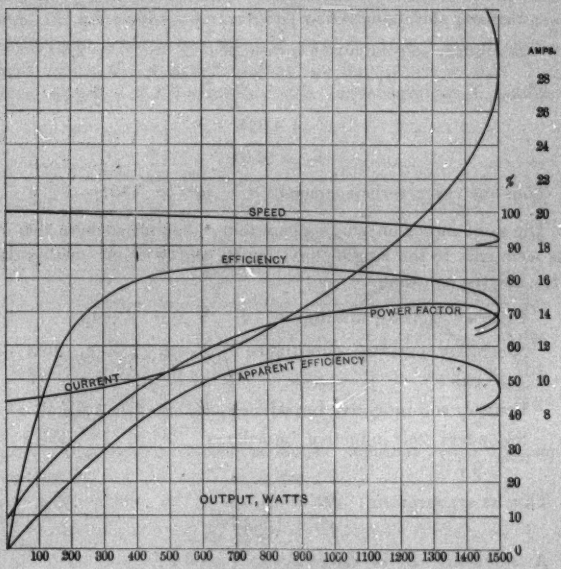

Fig. 4. Load Curves of Single-Phase Motor.

$$e_0 = 110.$$
$$Y_0{}^1 = .003 + .045\,j.$$
$$Y_1{}^1 = (1 - s)\,(.003 + .045\,j.)$$
$$Z_0{}^1 = .4 - 1.6\,j.$$
$$Z_1{}^1 = (1 + s)\,(.2 - .8\,sj).$$

parent efficiency, due to the primary circuit carrying the magnetizing current of both magnetic circuits. This is the more objectionable as single-phase circuits are mostly of high frequency, and the high-frequency induction motor has inherently a higher magnetizing current, in consequence of the lesser pitch

per pole, and thus lesser number of ampere-turns available in the motor field. This and the excessive current required with most starting devices, to get any noticeable starting torque, have thus far been the most serious objections to the introduction of the single-phase induction motor.

To improve the power factor, which as seen, in the motor in Fig. 4, reaches only 71% at full load and 53% at half load, condensers shunted across the motor terminals have been proposed, but have not been successful, for the following reasons:

At best the shunted capacity compensates for the lagging currents in the single-phase supply circuit only, but the excessive magnetizing current still exists in the motor circuits, causing a decrease of output, etc. In most of the high-frequency circuits existing to-day shunted capacity fails, however, entirely to reduce the current taken by a single-phase motor. To do this it is necessary that the E. M. F. wave of the generator resembles a sine wave with extreme closeness. This is not the case in most high-frequency systems, and is not desirable either, since the saw-tooth wave of the uni-tooth alternator, generally preferred, gives from 8 to 12% less hysteresis loss in the transformers than the sine wave, and thus a correspondingly higher all-day efficiency of the system.

On such circuits with saw-tooth E. M. F. waves the condenser, while taking away the wattless lagging current, adds triple and quintuple frequency currents of the same or even greater magnitude, and thus not raise but may even lower the power factor and the apparent efficiency.

Thus in the three-phase motor in Fig. 1, at 1500 watts output, the power factor is 78%, the current 6.6 amperes at 110 volts, hence:

$$\mathbf{I}_0 = 5.15 + 4.13\,j$$

and, to compensate for the lagging current, a condenser susceptance would be required:

$$k = \frac{4.13}{110} = .0375$$

Assuming the wave shape of impressed E. M. F., $e_0 = 110$ as:

$$\mathbf{E}_0 = 107_1 + 10_3 + 20_5 - 10_7$$

the current input in the condenser of general susceptance:

$$Y = -.0375\, n\, j_n$$

is:

$$\mathbf{I}_k = .03\, j_1 - 1.13\, j_3 - 3.75\, j_5 + 2.63\, j_7$$

thus: the total current:

$$\mathbf{I} = \mathbf{I}_0 + \mathbf{I}_k$$

$$= (5.15 + .10\, j_1) - 1.13\, j_3 - 3.75\, j_5 + 2.63\, j_7$$

or, absolute:

$$I = 6.99 \text{ amperes}$$

thus the power factor:

$$p = \frac{5.15}{6.99} = 73.7\%$$

that is, in this case, by the use of shunted capacity with a distorted wave of E. M. F., the power factor of the motor is reduced from 78% to 73.7%, due to the appearance of higher harmonics of current.

8. Investigations extending over some years have led me to an arrangement whereby the condenser can be utilized in the single-phase induction motor to compensate for the lagging current not only in the supply circuit but also in the motor proper, and that independently of the wave shape of impressed E. M. F., by utilizing the quadrature magnetic flux induced in the motor at speed.

If in a single-phase induction motor on the field or primary member, a second coil is arranged in quadrature position with the primary coil, this coil is in non-inductive relation to the primary coil, but E. M. F. is induced therein with the motor at speed by the secondary or armature, that is, this auxiliary coil is a tertiary circuit which receives its E. M. F. by double transformation.

Since, however, primary, secondary and tertiary have multitooth or distributed windings, and the magnetic flux in the secondary member revolves at constant intensity and constant velocity, due to its closed circuits of low resistance, the E. M. F. induced in the tertiary is an absolute sine wave irrespective of the wave shape of the E. M. F. impressed upon the primary (as fully borne out by tests). Connecting to the tertiary coil a condenser of the same condenser admittance as the secondary exciting admittance, the tertiary condenser circuit gives the

secondary magnetizing current, that is the secondary current becomes zero at synchronism, just as in a polyphase motor, and

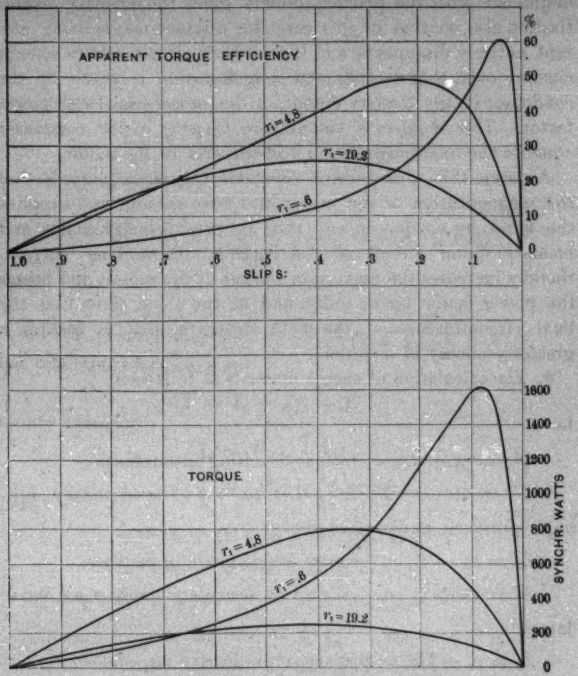

FIG. 5. Speed Curves of Single-Phase Motor.

$$e_0 = 110.$$
$$Y_0{}^1 = .003 + .045\,j.$$
$$Y_1{}^1 = (1 - s)\,(.003 + .045\,j.)$$
$$Z_0{}^1 = .4 - 1.6\,j.$$
$$Z_1{}^1 = (1 + s)\,(.2 - \left.\begin{matrix} \\ 1.6 - \\ 6.4 - \end{matrix}\right\} 1.6\,sj.)$$

the primary current is reduced to the magnetizing current of the main magnetic flux only.

If twice this capacity is used, the condenser current in the tertiary circuit supplies the magnetization of the quadrature

magnetic flux, and at the same time causes a current in the secondary to flow, which, carried by the rotation 90° or into alignment with the primary circuit, gives the primary magnetization also, so that in this case the primary magnetizing current entirely disappears, and the primary circuit carries energy current only, that is the total magnetization is given by the condenser in the tertiary circuit, or the motor has 100% power factor. This requires a volt-ampere capacity of the condenser equal to the total magnetizing volt-amperes of the motor.

As seen, this arrangement separates the energy supply, and the magnetization of the motor into two circuits and supplies the latter by condenser, and thus relieves primary mains and primary motor circuit of the lagging magnetizing current, thereby increases the maximum output of the motor, and brings the power factor up to unity, and at the same time uses the total circumference of the field structure, that is secures a greater economy of material.

9. The calculation of such a motor is as follows:

Let:

$Y_0^1 = g_0 + j\,b_0 =$ primary exciting admittance.

$Y_1^1 = (1 - s)\ Y_0^1 = (1 - s)\,(g_0 + j\,b_0) =$ secondary exciting admittance, as discussed above.

$Z_0^1 = r_0 - jx_0 =$ primary single-phase impedance.

$Z_1^1 = (1 + s)\ (r_1 - j\ s\ x_1) =$ secondary single-phase impedance.

$Z_2 = r_2 - j\ x_2 =$ tertiary self-inductive impedance.

$Y_3 = g_3 - j\ b_3 =$ condenser admittance, thus:

$Z_3 = r_3 + j\,x_3 = \dfrac{1}{Y_3} =$ condenser impedance, and

$Y_4 = \dfrac{1}{Z_2 + Z_3} = g_4 - j\,b_4 =$ total admittance of tertiary circuit.

Where: $s =$ slip.

Let, further:

$e =$ E. M. F. induced by the mutual magnetic main flux at full frequency.

Thus, $(1 - s)\, e$ = induced quadrature E. M. F.

It is then, at slip s:

tertiary current:

$$\mathbf{I}_4 = e\,(1 - s)\; Y_4$$
$$= e\,(1 - s)\,(g_4 - j\, b_4)$$

secondary exciting current:

$$\mathbf{I}_{10} = e\, Y_1^1 = e\,(1 - s)\,(g_0 + j\, b_0)$$

secondary load current:

$$\mathbf{I}_1 = \frac{e\, s}{Z_1^1} = \frac{e\, s}{(1 + s)\,(r_1 - j\, s\, x_1)} = e\,(a_1 + j\, a_2)$$

where:

$$a_1 = \frac{s\, r_1}{(1 + s)\,(r_1^2 + s^2\, x_1^2)} \qquad a_2 = \frac{s^2\, x_1}{(1 + s)\,(r_1^2 + s^2\, x_1^2)}$$

thus, total secondary current:

$$\mathbf{I}^1 = \mathbf{I}_1 + \mathbf{I}_{10} + \mathbf{I}_4$$

primary current:

$$\mathbf{I}_0 = \mathbf{I}^1 + e\; Y_0^1$$
$$= e\,(b_1 + j\, b_2)$$

where:

$$b_1 = a_1 + (2 - s)\, g_0 + (1 - s)\, g_4$$
$$b_2 = a_2 + (2 - s)\, b_0 - (1 - s)\, b_4$$

primary impressed E. M. F.:

$$\mathbf{E}_0 = e + Z_0^1\, \mathbf{I}_0$$
$$= e\,(c_1 + j\, c_2)$$

where:

$$c_1 = 1 + r_0\, b_1 + x_0\, b_2 \qquad c_2 = r_0\, b_2 - x_0\, b_1$$

thus:

$$e = \frac{e_0}{\sqrt{c_1^2 + c_2^2}} \text{ etc.}$$

torque:

$$T = e^2\, a_1\,(1 - s) \text{ etc.}$$

10. In practice the tertiary circuit is usually not arranged in quadrature position with the primary circuit, but under an angle of 60°, so as to give in the moment of starting same mutual in-

duction between primary and tertiary, and thereby induce at standstill an E. M. F. in the tertiary, which by causing a leading current in the condenser circuit, produces an auxiliary quadrature magnetic flux and thus makes the motor self-starting. At the same time, this arrangement offers the advantage of using the ordinary three-phase motor as singlephase condenser motor. It is shown in Fig. 6:

In a three-phase motor M, the terminals 1 and 2 are connected to the single-phase mains a and b, the terminals 1 and 3 to the condenser of admittance $Y^3 = g_3 - j\,b_3$. In this case the circuits 1–2 and 1–3–2 in multiple act as primary, and the circuits 1–3 and 1–2–3 in multiple as tertiary circuit. Since at present this arrangement, with either Δ or Y connected

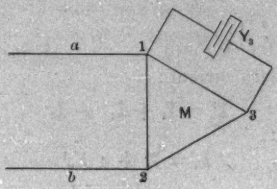

FIG. 6.

motor field, is exclusively used, and any other arrangement can be investigated in an analogous manner, I shall in the following restrict the discussion thereto.

With a position angle of 60° the tertiary circuit is still so far distant electrically from the primary that at speed it is practically independent of the wave shape of primary impressed E. M. F., and no serious effect is produced in the condenser circuit by a distortion of the primary wave shape. That is, with the saw-tooth wave of the uni-tooth alternator the compensation for lagging currents by the condenser is still practically complete and the power factor unity.

III.—THEORY OF SINGLE-PHASE CONDENSER MOTOR WITH THREE-PHASE ARRANGEMENT OF CIRCUITS.

11. Let, in a three-phase wound induction motor:

$Z_0 = r_0 - j\,x_0 =$ primary self-inductive impedance per motor delta circuit.

$Z_1 = r_1 - j\,x_1$ = secondary self-inductive impedance per circuit, reduced to primary

$Y_0 = g_0 + j\,b_0$ = primary exciting admittance, per motor delta circuit.

$Y_3 = g_3 - j\,b_3$ = admittance of condenser connected across motor terminals 1 and 3, Fig. 6 while motor terminals 1 and 2 are connected to the single-phase mains of E.M.F. e_0

On a three-phase system of E.M.F. e_0, it is, in starting, per motor circuit:

$$\mathbf{I}_1 = \frac{e}{r_1 - j\,x_1} = e\,(a_1 + j\,a_2) = \text{secondary current,}$$

where:

$$a_1 = \frac{r_1}{r_1^2 + x_1^2} \qquad a_2 = \frac{x_1}{r_1^2 + x_1^2}$$

e = counter E.M.F. of motor.

$e\,Y_0 = e\,(g_0 + j\,b_0)$ = primary exciting current, thus:

$$\mathbf{I}_0 = \mathbf{I}_1 + e\,Y_0 = e\,(b_1 + j\,b_2) = \text{primary current,}$$

where: $b_1 = a_1 + g_0 \qquad b_2 = a_2 + b_0$

thus, primary impressed E.M.F.:

$$\mathbf{E}_0 = e + Z_0\,\mathbf{I}_0 = e\,(c_1 + j\,c_2)$$

where: $c_1 = 1 + r_0\,b_1 + x_0\,b_2 \qquad c_2 = r_0\,b_2 - x_0\,b_1$

hence:

$$e = \frac{\mathbf{E}_0}{c_1 + j\,c_2}$$

and, absolute:

$$e = \frac{e_0}{\sqrt{c_1^2 + c_2^2}}$$

thus, total *stationary admittance*, per motor delta circuit:

$$Y = \frac{\mathbf{I}_0}{\mathbf{E}_0} = \frac{b_1 + j\,b_2}{c_1 + j\,c_2} = g + j\,b$$

where:

$$g = \frac{b_1\,c_1 + b_2\,c_2}{c_1^2 + c_2^2} \qquad b = \frac{b_2\,c_1 - b_1\,c_2}{c_1^2 + c_2^2}$$

the starting torque per circuit is:

$$T = e^2 a_1$$

$$= \frac{e_0^2 a_1}{c_1^2 + c_2^2}$$

thus, *total starting torque:*

$$T_\Delta = 3\ T = \frac{3\ e_0^2 a_1}{c_1^2 + c_2^2}$$

12. Starting as single-phase condenser motor, it is, in Fig. 6:

admittance of circuit from terminal 1 to 3: $Y + Y_3$

let the E.M.F. across Y_3 be: $\mathbf{E}_3$

admittance of circuit from terminal 3 to 2: Y

the E.M.F.: $e_0 - \mathbf{E}_3$

thus, since the same current passes both circuits in series:

$$Y + Y_3 \div Y = e_0 - \mathbf{E}_3 \div \mathbf{E}_3$$

or:

$$\mathbf{E}_3 = \frac{e_0\ Y}{2\ Y + Y_3} = e_0\ (h_1 + j\ h_2)$$

where:

$$h_1 = \frac{g\ (2\ g + g_3) + b\ (2\ b - b_3)}{(2\ g + g_3)^2 + (2\ b - b_3)^2},\quad h_2 = \frac{b\ (2\ g + g_3) - g\ (2\ b - b_3)}{(2\ g + g_3)^2 + (2\ b - b_3)^2}$$

and, absolute:

$$e_3 = e_0\ \sqrt{h_1^2 + h_2^2}$$

thus, the ratio of condenser E.M.F. to main E.M.F., at standstill:

$$h = \frac{e_3}{e_0} = \sqrt{h_1^2 + h_2^2}$$

The quadrature component of E.M.F. which gives the auxiliary quadrature or starting magnetic flux, is thus:

$$E_3' = e_0\ h_2$$

In the three-phase motor, at equality of the main and the quadrature fluxes, the quadrature flux is, as altitude of the equilateral triangle:

$$E^1 = \frac{e_0 \sqrt{3}}{2}$$

Thus, with single-phase condenser motor, in starting the ratio of auxiliary quadrature to main magnetic flux is:

$$t = \frac{E_3^1}{E^1} = \frac{2 h_3}{\sqrt{3}} = 1.155 h_3$$

and thus the *starting torque:*

$$T_0 = t T_\Delta = 1.155 h_3 T_\Delta$$

Since, in the circuit from terminal 3 to 2 the admittance $= Y$, the E. M. F. $e_0 - \mathbf{E}_3$, it is:
Current taken by the condenser motor in starting:

$$\mathbf{I}_0 = e_0 Y + (e_0 - E_3) Y$$

$$= \frac{e_0 Y (3 Y + 2 Y_3)}{2 Y + Y_3}$$

13. Running as single-phase condenser motor, it is, denoting the counter E. M. F. of the main magnetic circuit at slip s by e: secondary impedance:

$$Z_1^1 = \frac{Z_1}{3}(1 + s) = \frac{1 + s}{3}(r_1 - j s x_1)$$

primary impedance:

$$Z_0^1 = \frac{3}{4} Z_0 = \frac{3}{4}(r_0 - j x_0)$$

primary exciting admittance:

$$Y_0^1 = 1.5 Y_0 = 1.5 (g_0 + j b_0)$$

secondary exciting admittance:

$$Y_1^1 = 1.5 Y_0 [1 - (1 - t) s]$$

since for: $s = 1$, the secondary exciting admittance must be: $Y_1^1 = t Y_0^1$, t being the ratio of quadrature flux to main flux.
condenser admittance:

$$Y_3 = g_3 - j b_3$$

self-inductive impedance of tertiary circuit:

$$Z_2 = \tfrac{3}{4}\, Z_0 = \tfrac{3}{4}\,(r_0 - j\, x_0)$$

thus, total admittance of tertiary condenser circuit:

$$Y_4 = \frac{1}{Z_2 + \frac{1}{Y_3}} = g_4 - j\, b_4$$

The E. M. F. acting upon the condenser admittance Y_3 drops with increasing slip, from e_0 at $s = 0$ to $e_3 = e_0\, h$ at $s = 1$, thus is at slip s, approximately:

$$e_0\,[1 - (1 - h)\, s]$$

or inversely, assuming upon the condenser circuit instead of the induced E. M. F. e_3^1 of this circuit, the main induced E. M. F. e acting, the admittance has to be reduced in the proportion

$$\frac{e_3^1}{e},$$

that is, the apparent admittance of the condenser circuit at slip s, or admittance reduced to induced E. M. F. e, is:

$$Y_4^1 = Y_4\,[1 - (1 - h)\, s]$$

14. The calculation of the motor is now as follows: secondary load current:

$$\mathbf{I}_1 = \frac{s\, e}{Z_1\, s} = \frac{3\, s\, e}{(1 + s)\,(r_1 - j\, s\, x_1)} = e\,(a_1 + j\, a_2)$$

secondary exciting current:

$$\mathbf{I}_1^1 = e\; Y_1^1 = 1.5\; e\; Y_0\,[1 - (1 - t)\, s]$$

secondary condenser current:

$$\mathbf{I}_4 = e\; Y_4^1 = e\; Y_4\,[1 - (1 - h)\, s]$$

thus, total secondary current:

$$\mathbf{I}^1 = \mathbf{I}_1 + \mathbf{I}_1^1 + \mathbf{I}_4$$

primary exciting current:

$$\mathbf{I}_0^1 = e\; Y_0^1 = 1.5\; e\; Y_0$$

thus, total primary current:

$$\begin{aligned}\mathbf{I}_0 &= \mathbf{I}^1 + \mathbf{I}_0^1 \\ &= \mathbf{I}_1 + \mathbf{I}_4 + \mathbf{I}_1^1 + \mathbf{I}_0^1 \\ &= e\,(b_1 + j\,b_2)\end{aligned}$$

primary impressed E. M. F.:

$$\begin{aligned}\mathbf{E}_0 &= e + Z_0^1\,\mathbf{I}_0 \\ &= e\,(c_1 + j\,c_2)\end{aligned}$$

thus, main counter E. M. F.:

$$e = \frac{\mathbf{E}_0}{c_1 + j\,c_2}$$

or:

$$\mathbf{E} = \frac{e_0}{c_1 + j\,c_2}$$

and, absolute:

$$e = \frac{e_0}{\sqrt{c_1^2 + c_2^2}}$$

hence, primary current:

$$\mathbf{I}_0 = \frac{e_0\,(b_1 + j\,b_2)}{c_1 + j\,c_2} \qquad I_0 = e_0\,\sqrt{\frac{b_1^2 + b_2^2}{c_1^2 + c_2^2}}$$

volt-ampere input:

$$Q_0 = e_0\,I_0$$

power input:

$$P_0 = [\mathbf{I}_0\,e_0]^1 = e_0^2\,\frac{b_1\,c_1 + b_2\,c_2}{c_1^2 + c_2^2}$$

Fictitious torque (that is, torque, which would be given, if the quadrature flux equalled the main magnetic flux):

$$T^1 = e^2\,a_1 = \frac{e_0^2\,a_1}{c_1^2 + c_2^2}$$

At standstill, the torque must be:

$$T_0 = t\,T_\Delta$$

thus if:

$$T_0^1 = \left|\frac{e_0^2\,a_1}{c_1^2 + c_2^2}\right|_0 = \text{fictitious torque at standstill,}$$

denoting:

$$\frac{T_0}{T_0^1} = \left| \frac{t\, T_{\triangle}}{\dfrac{e_0^2\, a_1}{c_1^2 + c_2^2}} \right|_0 = v$$

it is:

torque at slip s:

$$T = T^1 [1 - (1 - v)\, s]$$

$$= \frac{e_0^2\, a_1}{c_1^2 + c_2^2} [1 - (1 - v)\, s]$$

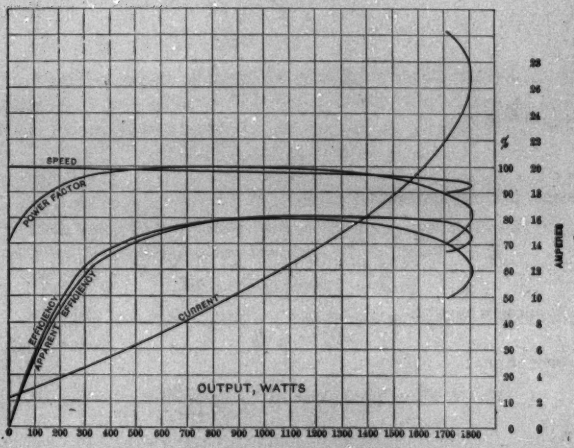

FIG. 7. Load Curves of Single-Phase Condenser Motor.

$$e_0 = 110$$
$$Y_0^1 = .003 + .045\, j.$$
$$Y_1^1 = (1 - 947\, s)\,(.003 + .045\, j.)$$
$$Z_0^1 = 4 - 1.6\, j.$$
$$Z_1^1 = (1 + s)\,(.2 - 8\, sj.)$$
$$Y_4^1 = (1 - .37\, s)\,(.0086 - .105\, j.)$$
$$T = e^2\, a_1\, (1 - 903\, s.)$$

and, power output:

$$P = T\,(1 - s)$$

$$= \frac{e_0^2\, a_1}{c_1^2 + c_2^2}\,(1 - s)\,[1 - (1 - v)\, s]$$

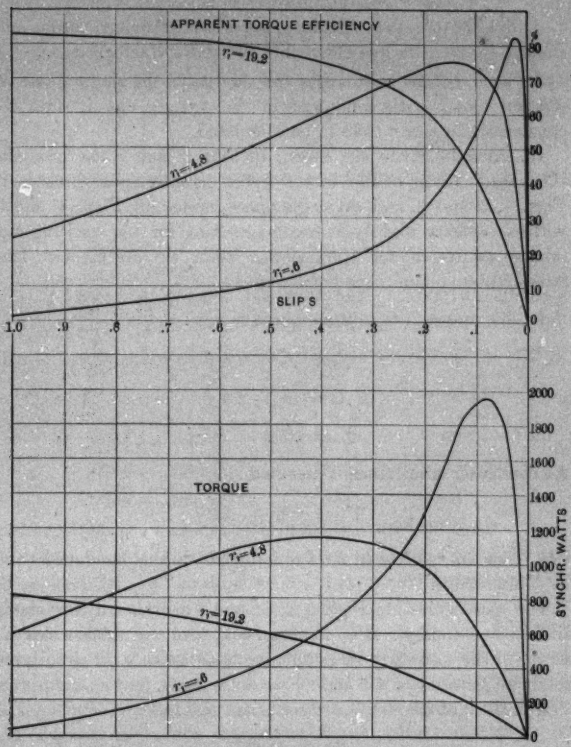

FIG. 8. Speed Curves of Single-Phase Condenser Motor.

$$e_0 = 110.$$

$$Y_0{}^1 = .003 + .045\,j.$$

$$Y_1{}^1 = (1 - .947\,s - .806\,s - .448\,s)(.003 + .045\,j.)$$

$$Z_0{}^1 = 4 - 1.6\,j.$$

$$Z_1{}^1 = (1 + s)(.2 - 1.6 - 6.4 -\} 1.6\,s\,j.)$$

$$Y_4{}^1 = (1 - .37\,s - .89\,s - .88\,s)(.0086 + .105\,j)$$

$$\text{Torque} \quad T = e^2\,a_1\,(1 - .903\,s)\;e^2\,a_1\,(1 - .65\,s)\;e^2\,a^1$$

And herefrom the efficiency, apparent efficiency, torque efficiency, apparent torque efficiency, and power factor.

As seen, before calculating the curves of the single-phase induction motor with condenser in the tertiary circuit, first the constants t, h, and v have to be calculated.

15. As an instance are shown in Figs. 7 and 8 the load and the speed curves of the motor given as three-phase motor in Figs. 1, 2, and 3, and as single-phase motor in Figs. 4 and 5, with a capacity slightly over-compensating for the exciting current so as to produce 100% power factor as intermediate load (about 1000 volt-amperes condensance).

In this motor, it is as three-phase motor:

$$Z_0 = .6 - 2.4\,j$$

$$Z_1 = .6 - 2.4\,j$$

$$Y_0 = .002 + .03\,j$$

As condenser admittance is assumed:

$$Y_3 = .003 - .09\,j$$

that is, an energy loss of 3.3% is assumed in the condenser and its compensator.

The speed curves in Figs. 2, 3, 5, and 8 are shown for short-circuited secondary: $r_1 = .6$ ohms, and also for resistances inserted in the secondary so as to give per secondary circuit the total resistance $r_1 = 4.8$ and: $r_1 = 19.2$ ohms.

It is then, at standstill as three-phase motor:

Three-phase:

$s = 1$ $\qquad e_0 = 110$

r_1:	I_0:	Y:	T_Δ:
.6	$5.4 + 22.4\,j$	$.049 + .204\,j$	825
4.8	$10.9 + 11.4\,j$	$.099 + .104\,j$	3030
19.2	$4.9 + 4.0\,j$	$.044 + .037\,j$	1480

In starting as single-phase condenser motor, it is:

$$Y_3 = .003 - .09\,j$$

thus:

r_1:	$\frac{Y}{2Y+Y_3}=h_1+jh_2$:	$t=1.155\ h_2$:	$h=\sqrt{h_1^2+h_2^2}$:	$T_0=tT_\Delta$
.6	$.63+.046j$	.053	.630	44
4.8	$.59+.168j$	.194	.613	590
19.2	$.40+.477j$	.552	.623	820

At standstill as single-phase condenser motor, for $s=1$, it is:

$$Y_4=.0086-.105j$$

thus:

Z_1^1:	Z_0^1:	Y_0^1:	$Y_1^1=t\ Y_0^1$:
$.4-1.6j$	$.4-1.6j$	$.003+.045j$	$.00016+.0024j$
$3.2-1.6j$	$.4-1.6j$	$.003+.045j$	$.0005+.0087j$
$12.8-1.6j$	$.4-1.6j$	$.003+.045j$	$.0017+.025j$

$Y_4^1=h\ Y_4$:	$T_0^1=e^2\ a_1$:	$v=\frac{T_0}{T_0^1}$
$.0054-.066j$	455	.097
$.0053-.064j$	1685	.35
$.0054-.065j$	820	1.00

and herefrom the motor is calculated, as explained above.

16. Comparing now the load curves of the motor under the three conditions of running, we see that as condenser motor on single-phase circuit the maximum output, 1810 watts, is higher than as ordinary single-phase motor, 1500 watts, but still very much less than as three-phase motor, 2680 watts. The slip at maximum output point is the same in the condenser motor as in the single-phase motor but less than in the three-phase motor. As single-phase motor the power factor, with a maximum of 73%, is less than as three-phase motor, with a maximum of 79%. As condenser motor, however, the power factor starts with 71.3% at standstill, reaches 100% at 800 watts and then decreases again to 90% at 1700 watts. In the total range from 260 watts to 1510 watts output the power factor is above 95%, that is practically unity. Below 800 watts output the current is leading, above it is lagging. In consequence of the high-power factor the apparent efficiency, that is the output per volt-ampere input is higher in the condenser motor than even in the three-phase motor, reaching a maximum of 80% in the former, and of only 69% in the latter. The efficiency is slightly less in the condenser motor, with a maximum of 81%, than in the single-phase

motor with 84% maximum. This is due to the energy loss in the condenser, or rather the compensator feeding the condenser (since with a 110-volt motor a compensator is required to bring the voltage at the condenser up to 500 volts as the minimum practicable in condenser). In the three-phase motor the efficiency is considerably higher, with 89% maximum.

It must be considered here, however, that these efficiencies, as

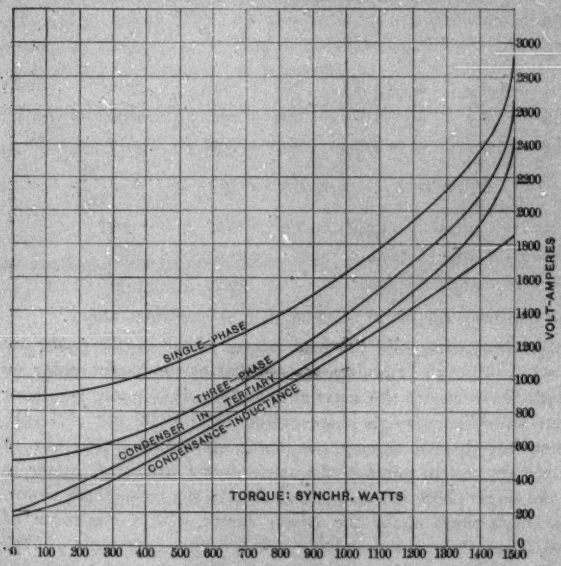

FIG. 9. Torque Curves.

Single-phase, $e_0 = 106$.
Three-phase, $e_0 = 77.5$
Single-phase, Condenser in Tertiary, $e_0 = 96$
Single-phase, Condensance-Inductance,, e_0 77.5

in my previous paper, do not include the friction loss of the motor, which, depending upon the individual conditions of the motor, as number of poles, speed, etc., cannot be included in a general theory; and the outputs given are thus output at the secondary conductors, or mechanical output plus friction.

Especially interesting is the current running light, or exciting current, which in the three-phase motor is 39.8% of the current at two-thirds maximum output, in the single-phase motor is 55.7%, but in the condenser motor only 17.4% of the current at two-third maximum output, which can approximately be considered as the proper full load rating. Figs. 2, 3, 5, and 8 show the torque and the apparent torque efficiency, that is the torque per volt-ampere input, as function of the speed of the motor. As seen, the condenser motor takes an intermediate position between the three-phase and the single-phase motor. In the three-phase motor the maximum torque remains the same with

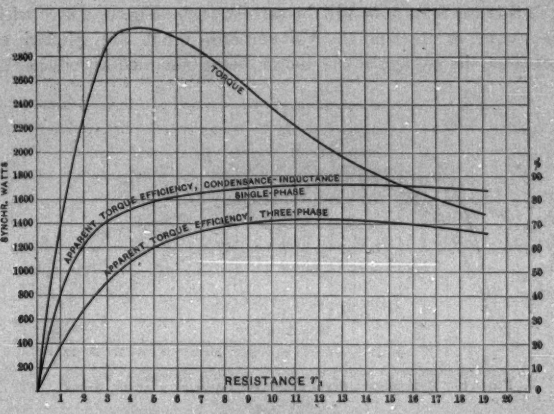

FIG. 10. Resistance Curves of Three-Phase Motor and Condensance-Inductance Single-Phase Motor.
$e_0 = 110$

increase of the secondary resistance, and the total torque curve merely shifts towards lower speed, the slip increasing proportional to the armature resistance. In the single-phase motor the maximum torque not only shifts towards lower speeds with increase of armature resistance, but falls off proportional to the slip, and thus rapidly becomes insignificant. In the condenser motor the falling off of the maximum torque, with increasing slip, by the introduction of secondary resistance, is much less, and the torque curves resemble more those of a three-phase

motor. Obviously the effect of the condenser on the starting torque is greatest with high secondary resistance, where the condenser current gets nearer in magnitude to the current taken by the motor proper. Especially interesting are the curves of apparent torque efficiency. While they fall off rapidly with the speed in the single-phase motor, they remain high in the condenser motor and the apparent torque efficiency of the latter, even in starting, is higher than with the three-phase motor. That is, with resistance in the secondary, the single-phase motor with condenser in the tertiary circuit can give a larger starting torque per volt-ampere input than the same motor as polyphase motor.

This latter feature is still better shown in Fig. 9, where with the torque as abscissæ the volt-amperes input are plotted as ordinates, the voltage at the motor being assumed as reduced to

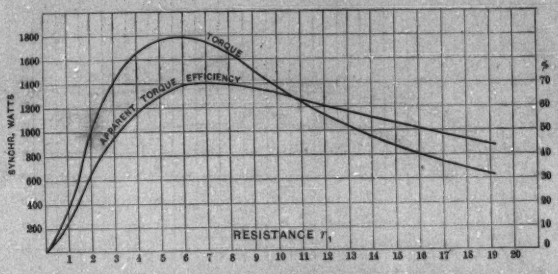

FIG. 11. Resistance Curves of Single-Phase Condenser Motor.

$$e_0 = 110$$
$$Y_s = .0034 - .2j$$

such a value as to give the same maximum torque of 1500 synchronous watts in all motors. This would require for the three-phase condition of operation 77.5 volts, single-phase 106 volts, and for the condenser motor 96 volts. These curves show that for any torque and especially for small values of torque, or light loads, the condenser motor takes the least, the single-phase motor the most volt-ampere input.

Curves 8 show, however, that while it is undesirable, with polyphase motors, to start with a short-circuited armature, that is, no rheostatic control, wherever any considerable starting torque is required, due to the excessive starting current required

under these conditions, this is still more the case with the condenser motor.

17. The variation of starting torque with the armature resistance is shown for the three-phase motor and for the condenser motor in Figs. 10 and 11, as calculated from the equations.

As seen, in both forms of motor with increase of resistance the starting torque increases, reaches a maximum, and then decreases again. The maximum in the three-phase motor is higher, 3030 synchronous watts, at lower secondary resistance, $r_1 = 4.5$ ohms, than with the condenser motor, where the maximum torque of 895 synchronous watts occurs at $r_1 = 6$ ohms. The apparent torque efficiency or starting torque per volt-ampere input reaches a maximum in the three-phase motor at far higher

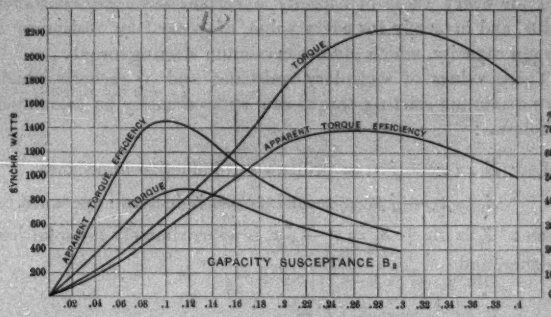

FIG. 12. Capacity Curves of Single-Phase Condenser Motor.

$$e_0 = 110$$
$$Y = .099 + .104\,j$$
$$.044 + .037\,j$$

resistance than the torque and is 72% at 12 ohms. In the condenser motor the maximum of apparent torque efficiency is 70.5%, and nearer the maximum torque, at 7 ohms. For the latter motor the starting conditions are calculated for a constant capacity of $Y_2 = (.017 - j)\ .2$, that is higher capacity than chosen in the motor in Figs. 7 and 8.

The effect on the starting torque of the condenser motor, of a variation of capacity is shown in Fig. 12 for the secondary resistances: $r_1 = 4.8$ and: $r_1 = 19.2$. As seen with the increase of capacity, in either case starting torque and apparent torque

efficiency increase, reach a maximum, and then decrease again. With higher armature resistance: $r_1 = 19.2$, the maximum starting torque is at $Y_3 = (.017 - j)\ .12$, and is 890 synchronous watts; with lower armature resistance: $r_1 = 4.8$, it is 1615 synchronous watts at $Y_3 = (.034 - j)\ .3$.

The cause of the falling off of the starting torque if the capacity is increased beyond a certain value is the decreasing voltage at the condenser. With the increase of capacity the total impedance of circuit 1-3 in Fig. 10 and thus the voltage decreases again, the circuit approaching the condition of short-circuit.

18. As seen in the preceding, the starting torque of the single-phase induction motor with condenser in the tertiary circuit depends upon the secondary resistance and upon the capacity, and for every value of capacity a certain secondary resistance exists, and inversely for every value of secondary resistance a certain capacity, which gives a maximum starting torque.

These values of secondary resistance and of capacity which give maximum starting torque can approximately be calculated as follows:

Let: $Z_0 = r_0 - j\,x_0 =$ primary self-inductive impedance,

$Z_1 = r_1 - j\,x_1 =$ secondary self-inductive impedance.

Then: $Z = Z_0 + Z_1 = r - j\,x$, is approximately the total impedance, and

$$Y = \frac{1}{Z} = g + j\,b = \frac{1}{r - j\,x}$$

is the total admittance of the motor at standstill.

Hereby the exciting current of the motor is neglected, which essentially amounts to assuming it as shunt to the total circuit instead of as shunt between primary and secondary circuit.

If $E_0 =$ impressed E. M. F., the counter E. M. F. is

$$E = E_0 \sqrt{\frac{r_1^2 + x_1^2}{r^2 + x^2}}$$

Neglecting the energy coefficient of the starting condensers, their admittances can be represented by:

$$Y_3 = -2\,j\,k$$

From the preceding calculation, the E. M. F. across the condenser is:

$$\mathbf{E}_2 = e_0 A,$$

where:

$$A = \frac{Y}{2Y + Y_2} = \frac{1}{2\left\{1 + \frac{Y_2}{2Y}\right\}} = \frac{1}{2(1 - kx - jkr)}$$

and the quadrature component of A is thus:

$$h_2 = \frac{kr}{2[(1-kx)^2 + k^2 r^2]}$$

the torque of the motor on three-phase circuit is:

$$T_\Delta = 3 e^2 a_1 = \frac{3 e^2 r_1}{r_1^2 + x_1^2}$$

thus, substituted for e:

$$T_\Delta = \frac{3 e_0^2 r_1}{r^2 + x^2}$$

and hence, the starting torque of the single-phase motor:

$$T = \frac{2}{\sqrt{3}} h_2 T_\Delta = \frac{\sqrt{3}\, e_0^2 k r r_1}{(r^2 + x^2)[(1-kx)^2 + k^2 r^2]}$$

$$= \frac{\sqrt{3}\, e_0^2 k r_1^2}{(r^2 + x^2)[(1-kx)^2 + k^2 r^2]} \left\{1 - \frac{r_0}{r}\right\}$$

Let $$f = \frac{T}{\sqrt{3}\, e_0^2} = \frac{k r r_1}{(r^2 + x^2)[(1-kx)^2 + k^2 r^2]}$$

$$= \frac{k r^2}{(r^2 + x^2)[(1-kx)^2 + k^2 r^2]} \left\{1 - \frac{r_0}{r}\right\}$$

or, approximately:

$$f = \frac{k r^2}{(r^2 + x^2)[(1-kx)^2 + k^2 r^2]}$$

The maximum starting torque, at given capacity k, as function of resistance r is determined by:

$$\frac{\delta f}{\delta r^2} = 0$$

which, expanded, gives:

$$x^2 (1 - k x)^2 - k^2 r^4 = 0$$

$$r = \frac{\sqrt{\pm x (1 - k x)}}{k} = \sqrt{\mp (x^2 - x/k)}$$

substituting these values of r into T, it is:

$$(1) \quad T_1 = \sqrt{3}\, e_0^2\, k \left(1 - \frac{r_0}{r}\right) = \sqrt{3}\, e_0^2\, k \left(1 - \frac{r_0}{\sqrt{x/k - x^2}}\right)$$

$$(2) \quad T_2 = \frac{\sqrt{3}\, e_0^2\, k \left(1 - \frac{r_0}{r}\right)}{(2\, k\, x - 1)^2} = \frac{\sqrt{3}\, e_0^2\, k}{(2\, k\, x - 1)^2} \left(1 - \frac{r_0}{\sqrt{x^2 - x/k}}\right)$$

The value of k giving maximum torque is found by:

$$(1) \qquad \frac{\delta T_1}{\delta k} = 0$$

or, expanded:

$$\left(\frac{1}{k} - x\right) = \frac{r_0}{x} \left(\frac{3}{2\, k} - x\right)^2$$

which gives the approximate solution, by substituting in the right hand side:

$$\frac{1}{k} = x:$$

$$\frac{1}{k} = x + \sqrt[3]{\frac{r_0^2\, x}{4}}$$

Instance:

$$e_0 = 110$$

$$x = 4.8 \qquad r_0 = .6$$

$$\frac{1}{k} = 5.556$$

$$r = \sqrt{\frac{x}{k} - x^2} = 1.905$$

$$r_1 = 1.305$$

$$Y_3 = -.36\,j$$
$$T_1 = 2580$$

(2) $$\frac{\delta T_2}{\delta k} = 0$$

or, expanded:

$$\left(x - \frac{1}{k}\right)^3 = \frac{r_0^2}{4x}\left(2x - \frac{1}{k}\right)^2$$

which gives the approximate solution, by substituting in the right hand side: $1/k = x$:

$$\frac{1}{k} = x - \sqrt[3]{\frac{r_0^2 x}{4}}$$

Instance: $e_0 = 110$

$x = 4.8$ $\qquad r_0 = .6$

$$\frac{1}{k} = 4.044$$
$$r = 1.905$$
$$r_1 = 1.305$$
$$Y_3 = -.495\,j$$
$$T_2 = 1870$$

19. From the equation:

$$T = \frac{\sqrt{3}\, e_0^2\, k\, r\, r_1}{(r^2 + x^2)\,[(1 - k\,x)^2 + k^2\, r^2]}$$

the maximum starting torque, for a given value of secondary resistance r_1, and thus total resistance r, as function of the capacity k, is found thus.

Let:

$$\varphi = \frac{T(r^2 + x^2)}{\sqrt{3}\, e_0^2\, r\, r_1} = \frac{k}{(1 - k\,x)^2 + k^2\, r^2}$$

it is then:

$$\frac{\delta \varphi}{\delta k} = 0$$

expanded, this gives:

$$1 - k^2\, x^2 - k^2\, r^2 = 0$$

or:

$$k = \frac{1}{\sqrt{r^2 + x^2}}$$

substituted:

$$T_0 = \frac{\sqrt{3}\, e_0^2\, r\, r_1}{2\, z^2\, (z - x)}$$

Instance:

$Z_0 = .6 - 2.4\,j \qquad Z_1 = 4.8 - 2.4\,j / 19.2 - 2.4\,j \qquad e_0 = 110$

thus: $\qquad Z = 5.4 - 4.8\,j / 19.8 - 4.8\,j$

$z = 1.225 \qquad / 20.37$

$k =$	.1385	.049
$Y_3 =$	$-.277\,j$	$-.098\,j$
$T_0 =$	2150	625

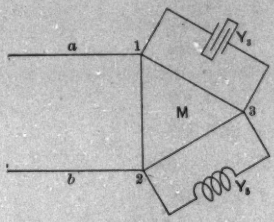

FIG. 13.

These values obviously are approximate only, for the reasons stated above.

IV. THREE-PHASE RELATION OF E. M. F.'S.

20. The starting torque of the three-phase wound single-phase induction motor can be increased far beyond the maximum value available by the use of capacity in the tertiary circuit, by shunting the third side of the motor triangle 3–2 by an admittance Y_5, which usually will have to be an inductance. By suitably varying the admittances Y_3 and Y_5, the voltages across 1–3 and 3–2 can be maintained equal, and equal to the impressed voltage across 1–2, that is, a perfect three-phase triangle produced.

Let thus, in Fig. 13:

$Y_3 = g_3 - j\,b_3 =$ condenser impedance shunted across circuit 1–3,

$Y_5 = g_5 + j\,b_5 =$ inductive impedance shunted across circuit 3–2, and

$Y = g + j\,b =$ total impedance per motor delta circuit.

It is then:

Impedance of circuits: 1–3: $Y + Y_3$

Impedance of circuits: 3–2: $Y + Y_5$

Thus, if $\mathbf{E}_3 =$ E.M.F. across 1–3, $\mathbf{E}_5 =$ E.M.F across 3–2, it is:

$$\mathbf{E}_3 \div \mathbf{E}_5 = Y + Y_3 \div Y + Y_5$$

since the same current passes through both circuits in series.

The condition of exact three-phase relation of the three E.M.F.'s at the motor terminals is, that $\mathbf{E}_3$ and $\mathbf{E}_5$ are equal, and 120° displaced in phase, that is:

$$\frac{\mathbf{E}_3}{\mathbf{E}_5} = \cos 120° + j \sin 120°$$

$$= \frac{-1 + j\sqrt{3}}{2}$$

thus, substituted:

$$\frac{Y + Y_3}{Y + Y_5} = \frac{-1 + j\sqrt{3}}{2}$$

Assuming now the energy loss in the inductance Y_5 as 5%, that in the condensance Y_3 as 1.7%, it is:

$$Y_5 = b_5\,(.05 + j)$$

$$Y_3 = b_3\,(.017 - j)$$

Substituting these values, and $Y = g + j\,b$, in the equation:

$$\frac{Y + Y_3}{Y + Y_5} = \frac{-1 + j\sqrt{3}}{2}$$

separating the real and the imaginary components, and resolving, gives:

$$b_3 = 1.885\,g + .95\,b$$

$$b_5 = 1.836\,g - 1.012\,b$$

thus:

$$Y_3 = (.017 - j)\,(1.885\,g + .95\,b)$$
$$Y_5 = (.05 + j)\,(1.836\,g - 1.012\,b)$$

and, the total current input in the motor:

$$\mathbf{I}_0 = e_0\,Y + \frac{e_0}{2}(1+j\sqrt{3})\,(Y+Y_3) = e_0\,Y + \frac{e_0}{2}(1-j\sqrt{3})(Y+Y_5)$$

$$= e_0\left\{(3.14\,g - .035\,b) + j\,(1.038\,b - .04\,g)\right\}$$

Neglecting, as approximation, the energy components of Y_3 and Y_5, it is:

$$Y_3 = -j\,b_3$$
$$Y_5 = j\,b_5$$

thus, substituted:

$$b_3 = g\sqrt{3} + b$$
$$b_5 = g\sqrt{3} - b$$

and:

$$\mathbf{I}_0 = e_0\,(3\,g + j\,b)$$

while in the three-phase motor the total current is:

$$\mathbf{I}_0 = 3\,e_0\,(g + j\,b)$$

that is, two-thirds of the wattless current is compensated for.

Or in other words, by using the two impedances Y_3 and Y_5 of the values calculated above, on a single-phase circuit, a three-phase relation of E. M. F.'s is produced, that is, the motor operates as three-phase motor with the same maximum output, torque, etc., but with a lesser current input, that is higher power factor, the wattless component of current being reduced to one-third. That is in the two sides of the E. M. F. triangle produced by the capacity inductance device, the total magnetizing current is supplied by the capacity, and the supply current gives the magnetizing current of one-phase only.

Obviously to maintain three-phase relation of E. M. F.'s, the susceptances b_3 and b_5 have to be changed with the speed as the above given functions of b and g.

Thus from the values of power factor and current and E. M F., calculated for the motor as three-phase motor, the total admittance of the motor:

$$Y = \frac{\mathbf{I}_0}{\mathbf{E}_0} = \frac{(b_1\,c_1 + b_2\,c_2) + j\,(b_2\,c_1 - b_1\,c_2)}{c_1^2 + c_2^2}$$

is calculated as function of slip s, and herefrom the impedances Y_3 and Y_5 as functions of the slip s required to give a three-phase triangle and the current input of the motor calculated:

The load curves of the single-phase motor with two impedances Y_3 and Y_5 adjusted so as to give a three-phase triangle are shown

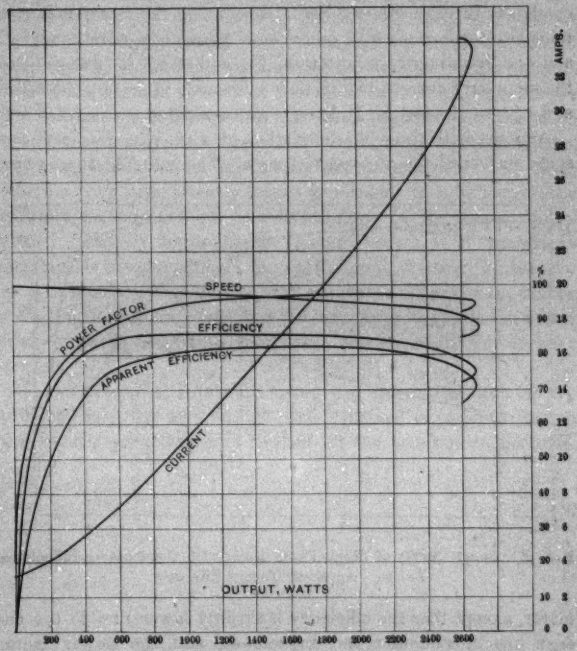

FIG. 14. Load Curves of Single-Phase Motor with Condensance-Inductance Device.

$$e_0 = 110$$

in Fig. 14, the speed curves in Fig. 2 and 15, and the volt-ampere input as function of the torque, reduced to the same maximum torque of 1500 synchronous watts as the other motors, that is, to 77.5 volts at the terminals, is given in Fig. 9.

In this case, where exact three-phase relation is produced, the

motor can be calculated as three-phase motor, with the only exception of the current input $\mathbf{I}_0$, which is given by above formula, and the quantities dependent thereupon as Q_0, P_0, etc.

Where no three-phase relation is produced, but for instance fixed amounts of capacity and of inductance inserted, the calculation is the same as in the chapter on the single-phase motor with condenser in the tertiary circuit, except that the motor now contains two tertiary circuits, one closed by capacity admittance Y_3 and one by inductive admittance Y_5, and thus in determining the secondary current two tertiary currents $e\,[1 - (1 - s)\,h_3]\,Y_4$ and $e\,[1 - (1 - s)\,h_5]\,Y_6$ have to be considered.

As seen, with three-phase relation of E. M. F.'s the load and speed curves of the motor are those of the motor as three-phase

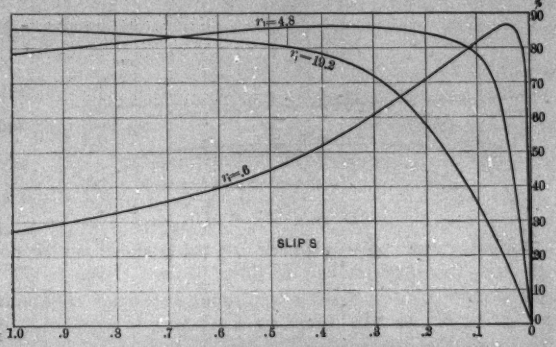

FIG. 15. Speed Curves of Single-Phase Motor with Condensance-Inductance Device. Apparent Torque Efficiency.

motor, except that the efficiency is slightly lower due to the energy loss in condenser and inductance, and the power factor greatly raised, from a maximum of 79% to a maximum of 97%, or almost unity. As a result, the apparent efficiency and the apparent torque efficiency are far higher than those of the three-phase motor and higher still than those of the single-phase condenser motor at constant capacity in the tertiary circuit. Or in other words, this motor requires the lowest volt-ampere input of all, for a given torque.

The impedance Y_3 is always a capacity. The impedance Y_5 is with short-circuited low resistance armature as shown in Fig.

15, a capacity from standstill up to a certain speed, then becomes an inductance, and very close to synchronism again a capacity. With high resistance armature or rheostatic control it is always an inductance, except very close to synchronism.

It thus follows, that in single-phase induction motors with a condenser in the tertiary circuit, compensating or slightly over-compensating for the exciting current at load, an apparent torque efficiency or torque per volt-ampere input can be produced equal or higher than that of the same motor on polyphase circuits, and with rheostatic control of the secondary, a starting torque per volt-ampere fully equal to that of the motor under the same conditions on polyphase circuit. The maximum starting torque, however, which can be produced thereby is limited, and thus, where a very large starting torque is needed, in addition to the capacity, an inductance in the third phase of the motor is required. Hereby, by producing an equilateral E. M. F. triangle in starting, in the single-phase motor, the same starting torque can be produced as given by this motor on three-phase circuit, that is, a starting torque far exceeding the maximum torque of the motor when running single-phase or with condenser in the tertiary circuit.

In the preceding I have discussed the theoretical side of the problem only, but may remark that practical experience during the last few years has fully proved all the conclusions deduced.

Schenectady, N. Y.
January, 1900.

A paper presented at the 170th Meeting of the American Institute of Electrical Engineers, New York, January 24th, 1900. President Kennelly in the Chair.

[ADVANCE COPY—SUBJECT TO REVISION.]

A NEW METHOD OF TRACING ALTERNATING CURRENT CURVES.

BY FITZHUGH TOWNSEND.

The following method of determining the instantaneous values of alternating current waves recommends itself by its simplicity, accuracy, and rapidity. In addition to this, the method is capable of determining directly a curve of induction.

Fig. 1 shows the connections for taking the curves of electromotive force and magnetic induction simultaneously. E is the E. M. F., the curve of which is to be determined, R_1, R_2 and R_3 are ohmic resistances, e is a small transformer with open magnetic circuit, and C is the contact maker. This consists of a disk, the perimeter of which, in a bipolar alternator, is divided into two sections. One section only is conducting, and since the disk is mounted on the shaft of the machine, or on that of a synchronous motor, the current can flow in the circuit of the contact maker only during one-half of a period. d is a Weston portable voltmeter, connections being made directly to the movable coil.

Determination of a Curve of Induction.

The double throw switch b shown in Fig. 1 should be closed so as to connect points 3 and 4 to 5 and 6, switch a being open. The required curve can then be obtained by noting the deflections of the instrument, d, corresponding to successive angular positions of the contact brush f. The theory on which this is based is as follows: Owing to the construction of the contact maker, the current i, which passes through the instrument d, flows during

one-half of a period only. During the active half period, however, i is proportional to E, the electromotive force: i will therefore be an interrupted current having the form of the electromotive force as shown in Fig. 2. The position of the

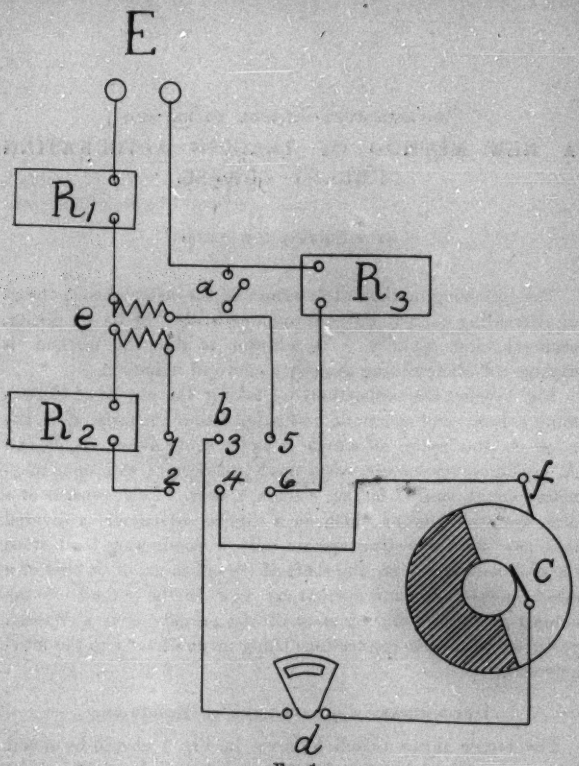

FIG. 1.

time interval $t_2 - t_1$ will depend on the angular position of the contact brush f.

The deflection of the voltmeter will be proportional to the number of impulses per second. Each impulse being proportional to the time integral of the current during the interval $t_2 - t_1$.

Therefore,

$$\theta = K\int_{t_1}^{t_2} i\,dt = K_1\int_{t_1}^{t_2} E\,dt = K_2\int_{t_1}^{t_2}\frac{d\,B}{d\,t}\,dt = K_2(B)_{t_1}^{t_2} = K_2\,B$$

If the brush f, and consequently the interval $t_2 - t_1$, are moved through a period, and the values of θ at different points noted, the curve of the induction B will be obtained.

Determination of a Curve of Electromotive Force.

The switch b should now be closed so as to connect points 3 and 4 to 1 and 2. The contact maker and the voltmeter will

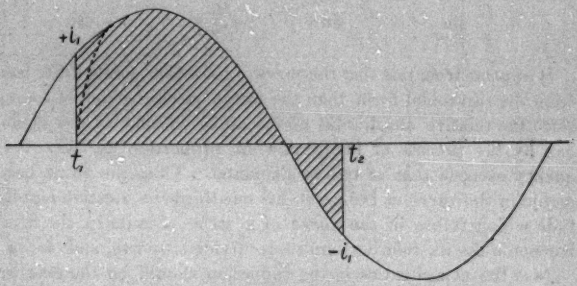

Fig. 2.

then be in the secondary circuit of the small transformer e. Close switch a. The curve of the electromotive force may then be obtained, as in the case of the curve of induction, by noting the deflections of the voltmeter for different angular positions of the brush f. It can be shown readily that this is the case. If the constants of the transformer e are suitably related to each other, the primary current will be the same in form as the electromotive force E, and practically independent of the value of the secondary current i. This will be brought about when the electromotive force of mutual induction in the primary, due to the secondary current is only a small percentage of the E. M. F., E. We will then have

$$i = K\frac{dE}{dt} \text{ and } \theta = K_1\int_{t_1}^{t_2}\frac{dE}{dt}\,dt = K_2(E)_{t_1}^{t_2} = K_3 E$$

Fig. 3 shows curves of B and E in a transformer obtained as described above. The form of these curves requires a few words of comment.

Any complex harmonic electromotive force generated by a properly designed alternator, may be represented by the equation $E = A_1 \sin p\,t + A_3 \sin 3\,p\,t + A_5 \sin 5\,p\,t + \text{etc.} + B_1 \cos p\,t + B_3 \cos 3\,p\,t + B_5 \cos 5\,p\,t + \text{etc.}$ Therefore,

$$B = \int E\,dt = -\left(\frac{A_1}{p}\cos p\,t + \frac{A_3}{3p}\cos 3\,pt + \frac{A_5}{5p}\cos 5\,p\,t + \text{etc.}\right)$$
$$+ \frac{B_1}{p}\sin p\,t + \frac{B_3}{3p}\sin 3\,p\,t + \frac{B_5}{5p}\sin 5\,p\,t + \text{etc.}$$

It appears from this that the curve of induction should differ less from the sinusoidal form than the curve of electromotive force, since the relative amplitudes of the upper harmonics are made less by the process of integration in proportion as their frequency exceeds that of the fundamental. Thus the third harmonic in the curve of induction has one-third the relative amplitude which it has in the curve of E. M. F. Similarly, the fifth harmonic has its relative amplitude divided by five, and so on.

Also the complexities in the induction should be the reverse of those in the electromotive force. In other words, a peak in curve B should correspond to a depression in curve E.

In order to prove the accuracy of the method, and to show that the curve B in Fig. 3 is correct, the true curve of induction was constructed from the electromotive force curve by a process of integration. The constructed curve was found to coincide with the experimental curve as shown.

The method of obtaining the curve of induction from the curve of E. M. F. by integration, is extremely laborious. The method described in this paper is the only one to-day which is capable of determining the curve of induction directly, thereby saving much time and trouble.

In determining the curve of induction, the phase depends on the relative values of the resistance and inductance of the circuit through the interrupter. A variation of the amplitude of

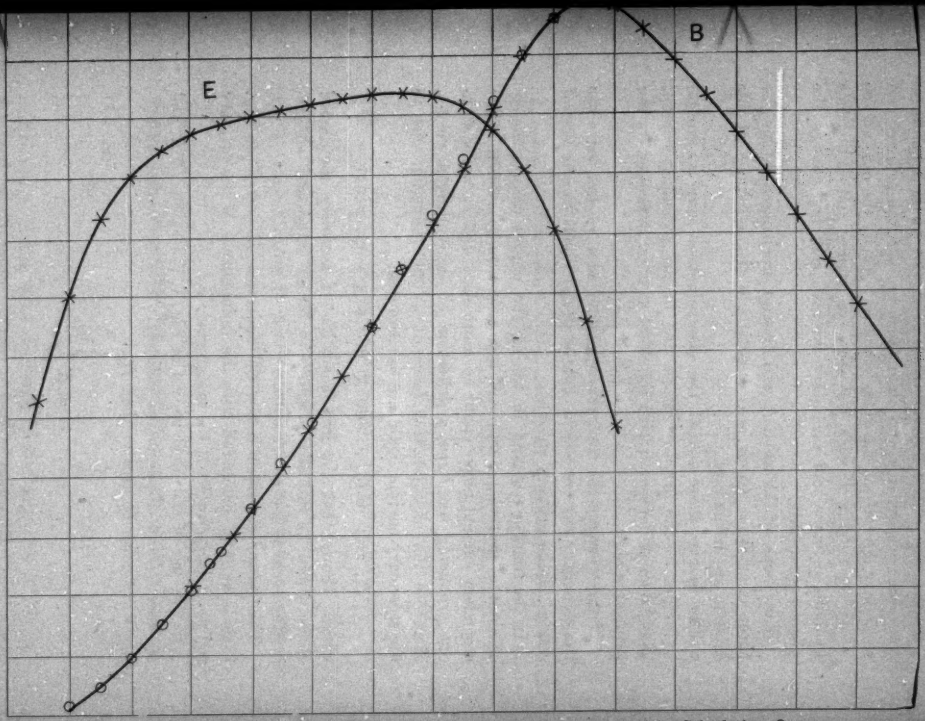

FIG. 8. Points Determined Experimentally, ×. Points Determined by Calculation, O.

E will not change the phase of the curve to a perceptible extent.

In the case of the determination of E, the phase depends on the values of resistance and inductance in both the primary and secondary circuits of the transformer e. As in the case of the B curve, varying E with the same resistance and inductance in both primary and secondary circuits, will not change the phase appreciably.

The shape of the wave E will remain unchanged, so long as three conditions which are essential to the accuracy of the method are maintained.

The first of these is that the electromotive force of mutual induction in the primary circuit due to the secondary current shall be small compared with the total E. M. F. impressed on the primary circuit.

If this were not the case, the secondary E. M. F., and therefore the secondary current would not be proportional to the rate of change of the primary current.

In the transformer used in taking the curves shown, the coefficient of mutual induction M was found to be .00076. The secondary current never could be more than 0.03 amperes without producing too large a deflection of the voltmeter needle, therefore the electromotive force of mutual induction due to the secondary current could not be more than

$$E_m = p\, M\, C_2 = 760 \times .00076 \times 0.03 = .017,$$

which shows that the first condition of accuracy mentioned above would be fulfilled to within 1% for an impressed electromotive force as low as 1.7 volts.

The second condition is that the resistance R_1 and R_2 in Fig. 1 shall be great compared with the inductances of the primary and secondary circuits of the transformer e.

In the case of the curve shown in Fig. 4, the values of R_1 and R_2 were 30 and 50 ohms respectively. If E is a complex harmonic, R_2 must be greater than R_1.

Since

$$i = K \frac{d\,E}{d\,t},$$

the upper harmonics in the secondary current i will be much more pronounced than they are in the curve E.

If R_2 were not large compared with the secondary inductance, the form of the current i might be quite different from that which it should have in order to give accurate results.

The third condition which must be fulfilled in order that the curve obtained may be correct in shape, is that the time constant of the secondary circuit shall not exceed a certain value.

When a circuit containing inductance is closed, the current

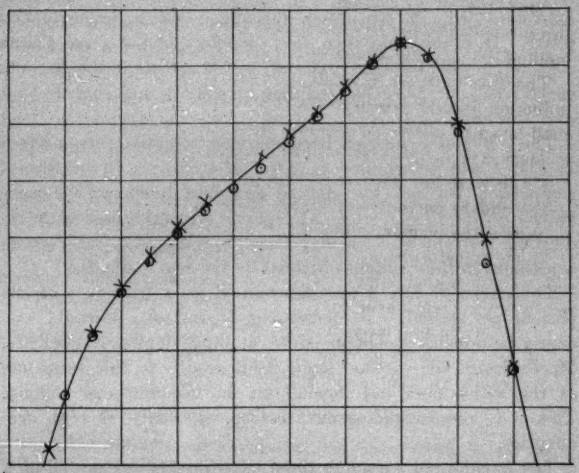

FIG. 4. Points Obtained by Zero Method, ⊙. Points Obtained by Present Method, ×.

does not reach its value E/R until a certain time has elapsed, and during this interval the current is

$$i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

Evidently if the time constant is too large, or if R/L is too small, the integral of the secondary current will not be the entire shaded area of Fig. 2, but will be bounded at the make

by some curved line instead of a perpendicular, as shown by the dotted line in the figure. This would be a source of error, which would come in more and more as the secondary current integral drew near to its zero value. When this integral is a maximum, the make takes place at $i = 0$, hence at this point of the curve the effect would not occur. The result of the time constant of the circuit would therefore be a decrease in the relative size of the ordinates near the zero points, and also a slight shifting of the zero points with respect to the maximum, the maximum ordinate remaining unchanged both in intensity and phase. It is probable, then, that the effect of too great a time constant in the secondary circuit would result in making the curve come out more pointed than it really is, and slightly bent over sideways.

The curve of a complex harmonic electromotive force shown in Fig. 4 was determined as a test of accuracy: It furnishes a comparison between the method described here, and the well-known zero method in which a telephone receiver is used as the indicating instrument. As may be seen, there is a satisfactory agreement in the readings obtained by the two methods.

The curve of Fig. 4 was determined at a frequency of 120 periods per second. By increasing R_1 and R_2, it would evidently be possible to obtain curves at much higher frequencies. Mechanically, the method lends itself readily to this, inasmuch as the action does not depend on an instantaneous contact. This is a very advantageous feature, especially at high frequencies.

The transformer e used in these experiments was wound on a wire core $1\frac{1}{2}''$ in diameter. There were 100 turns both in the primary and secondary windings.

The following are the principal constants of this transformer:

L_1 (on open secondary)	.00207 henrys.
R_1 " " "	.179 ohms.
L_2 " " primary	.00152 henrys.
R_2 " " "	.096 ohms.
M	.00076 henrys.
Resistance of voltmeter	.87 ohms.
Inductance " "	.00034 henrys.

Of course, only relative values are given by the readings of the voltmeter. In order to obtain absolute values, some means of calibration must be adopted. The easiest way to accomplish this is to switch the apparatus in series with some electromotive

force, either direct or alternating, the maximum value of which is known. The deflection obtained divided by the true value of the electromotive force it represents will be the constant of the instrument for the particular amounts of resistance and self-induction which may be connected in the primary and secondary circuits of the transformer e.

The fact that the brush f makes contact for a half period and not for a very small fraction of a period, is a great advantage, inasmuch as the difficulties of a method of instantaneous contact are usually due to the short time of contact, making it hard to keep the apparatus in good working order. With the interrupter described here, however, there is no more difficulty than with an ordinary direct current commutator, and in fact rather less, as sparking, which is always the main evil of the commutator is practically absent in the interrupter.

In addition to the above, the present method seems to be the only one by which the curves of magnetic induction or quantity of electricity can be determined simultaneously with those of electromotive force and current.

Alternating Current Laboratory,
Electrical Engineering Dept.,
Columbia University,
New York City, January, 1900.

Date of Annual and General Meetings.

At the meeting of the Executive Committee, on February 28th, it was decided that the Annual Business Meeting of the Institute for the election of officers, and reception of the yearly reports, would be held in New York City, on Tuesday, May 15th, being the third Tuesday in the month, as required by the Constitution.

The General Meeting for the reading and discussion of professional papers will begin at Philadelphia, in the afternoon of the following day, Wednesday, May 16th, and continue for two or three days, as may be determined upon hereafter. The meeting will then adjourn to Paris, August 16th, 1900.

DIED.

Rittenhouse:—At Denver, Colo., February 26th, 1900, Charles T. Rittenhouse, eldest son of M. and R. L. Rittenhouse of New York City. Mr. Rittenhouse was a graduate of Columbia, and formerly on the editorial staff of the *Electrical World*. He was elected an Associate Member of the Institute, February 21st, 1894.

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